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STRATEGIC DIVISION

Mathematical Models for Ground Combat

by

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April 1957

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ANALYTICAL "CALLING THE RESERVE" PROBLEM: BALANCE OF SPEED VS VULNERABILITY

by

George A. Gamow and Richard E. Zimmerman

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Dated 13 January 1952

ANALYTICAL "CALLING THE RESERVE" PROBLEM
(Balance of Speed vs. Vulnerability)

One of the important features of the Monte Carlo war-games problem is the calling of reserves by the units which encounter superior enemy forces. This will be accomplished by building into the strategy of the game, a special proviso according to which friendly forces located in the vicinity of the unit calling for help will get a preferential motion towards the trouble spot. Whereas, in the case of complicated situations which may be encountered in such war games the analysis can be carried out only by Monte Carlo method, it is interesting to consider some highly simplified cases which permit purely analytical solutions.

We take here, as an example, a linear case in which two opposing forces send out a given fraction α of their total strength to meet the enemy at a point located half way between them. It will be assumed that the opposing forces comprise equal number of units, but that, whereas the force H consists of slow heavily armored units, the units forming the force L are faster but carry less armor. Thus,

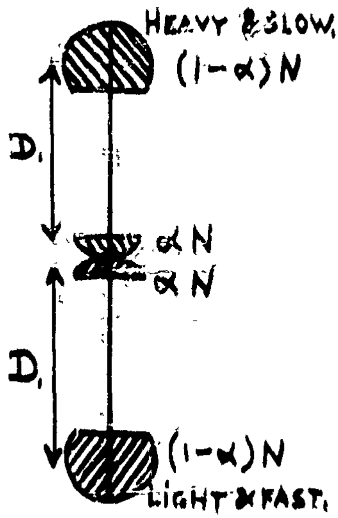

Fig. 1

if V_h and V_l are the speeds of heavy and light units we have

$$\frac{V_l}{V_h} > 1 \qquad (1)$$

The effect of armor (and armament) is characterized by the numerical values of the coefficients $\underline{K_h}$ and $\underline{K_l}$ in the Lanchester equations:

$$-\frac{dh}{dt} = K_l \cdot l \qquad (2a)$$

$$-\frac{dl}{dt} = K_h \cdot h \qquad (2b)$$

where $\underline{h}$ and $\underline{l}$ denote the number of heavy and light units participating in any given engagement. We take:

$$\frac{K_l}{K_h} < 1 \qquad (3)$$

We will assume that when the advanced units of L-forces encounter the advanced units of H-forces, and find themselves in a losing position (because $K_l \ll K_h$), light reserve (the entire remaining force) will be immediately called in. In respect to H-forces we will consider two extreme cases:

(a) <u>Complete intelligence</u> concerning the movements of L-reserves. In this case H-reserves will be called in simultaneously with L-reserves, but will arrive somewhat later because of their lower speeds.

(b) <u>Complete lack of intelligence</u>, in which case H-reserves will be sent in only when L-reserves arrive on the battlefield.

Thus, whereas L-reserves will always arrive at the time $t_1 = D/V_l$ where D is the distance of the reserves from the

battlefield, H-reserves will arrive at times:

$$t_2 = \frac{D}{V_h} \qquad (4a)$$

or

$$t_2' = \frac{D}{V_l} + \frac{D}{V_h} \qquad (4b)$$

depending on H-intelligence. If after the arrival of H-reserve the total number of H and L units will satisfy the Lanchester's balance condition, both forces will continue to lose equally percentage wise, and the battle will become a draw. We want to find the relations between the K_l/K_h and V_l/V_h ratios which would lead to a draw for various values of α and D.

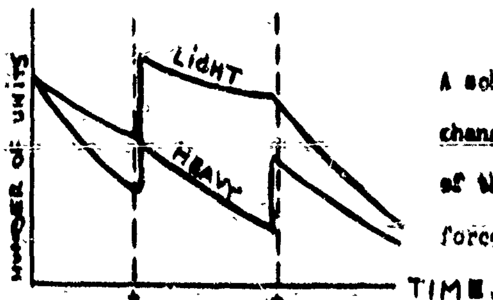

Fig. 2

A schematic picture of the change in the numbers h and l of the battling heavy and light forces as given in Fig. 2.

Differentiating each of the equations (2) with respect to time and inserting the other we obtain:

$$\frac{d^2h}{dt^2} = K_l \cdot K_h \cdot h = K^2 h \qquad (5a)$$

$$\frac{d^2l}{dt^2} = K_l \cdot K_h \cdot l = K^2 l \qquad (5b)$$

where

$$K = \sqrt{K_h \cdot K_l}$$

If we measure time in K^{-1} units, which can be called one clash equations (5ab) becomes:

$$\frac{d^2h}{d\tau^2} = h \qquad (\tau = t\cdot\sqrt{K_l\cdot K_h}) \tag{6a}$$

$$\frac{d^2l}{d\tau^2} = l \tag{6b}$$

and have the solutions:

$$h = a e^{-\tau} + b e^{+\tau} \tag{7a}$$

$$l = c e^{-\tau} + d e^{+\tau} \tag{7b}$$

Where four coefficients must be determined by the initial conditions.

At $\tau = 0$ we have:

$$h_0 = a + b = \alpha N \tag{8a}$$

$$l_0 = c + d = \alpha N \tag{8b}$$

$$\left(\frac{dh}{d\tau}\right)_0 = -a + b = -\beta^{-1}\alpha N \tag{8c}$$

$$\left(\frac{dl}{d\tau}\right)_0 = -c + d = -\beta\alpha N \tag{8d}$$

where:

$$\beta = \sqrt{\frac{K_h}{K_l}} \tag{9}$$

Determining four coefficients we get:

$$a = \frac{1}{2}\alpha N\left(1 + \beta^{-1}\right) \tag{10a}$$

$$b = \frac{1}{2}\alpha N\left(1 - \beta^{-1}\right) \tag{10b}$$

$$c = \frac{1}{2}\alpha N(1+\beta) \qquad (10c)$$

$$d = \frac{1}{2}\alpha N(1-\beta) \qquad (10d)$$

Thus, immediately after the arrival of light reserves:

$$h_1 = \frac{1}{2}\alpha N\left\{(1+\beta^{-1})e^{-T_1} + (1-\beta^{-1})e^{T_1}\right\} \qquad (11a)$$

$$\ell_1 = \frac{1}{2}\alpha N\left\{(1+\beta)e^{-T_1} + (1-\beta)e^{T_1} + 2\frac{1-\alpha}{\alpha}\right\} \qquad (11b)$$

The Lancaster equations are now applied to the time interval $T_1 \longrightarrow T_2$. We use constants a,b,c,d over again writing:

$$h_1 = a + b \qquad (12a)$$

$$\ell_1 = c + d \qquad (12b)$$

$$\left(\frac{dh}{dT}\right)_1 = -a + b = -\beta^{-1}\ell_1 \qquad (12c)$$

$$\left(\frac{d\ell}{dT}\right)_1 = -c + d = -\beta h_1 \qquad (12d)$$

Solving these equations we get:

$$a = \frac{1}{2}(h_1 + \beta^{-1}\ell_1) \qquad (13a)$$

$$b = \frac{1}{2}(h_1 - \beta^{-1}\ell_1) \qquad (13b)$$

$$c = \frac{1}{2}(\ell_1 + \beta h_1) \qquad (13c)$$

$$d = \frac{1}{2}(\ell_1 - \beta h_1) \qquad (13d)$$

Thus at $T = T_2$, after arrival of H-reserves, we obtain:

$$h_2 = \frac{1}{2}\left\{h_1\left[e^{-(T_2-T_1)} + e^{(T_2-T_1)}\right] + \beta^{-1}\ell_1\left[e^{-(T_2-T_1)} - e^{(T_2-T_1)}\right]\right\} + (1-\alpha)N \qquad (14a)$$

$$\ell_2 = \frac{1}{2}\left\{\ell_1\left[e^{-(T_2-T_1)} + e^{(T_2-T_1)}\right] + \beta h_1\left[e^{-(T_2-T_1)} - e^{(T_2-T_1)}\right]\right\} \qquad (14b)$$

The condition for balance for $\mathcal{T} \geq \mathcal{T}_2$ are:

$$\left(\frac{1}{h_2}\frac{dh_2}{d\mathcal{T}}\right)_{\mathcal{T}\geq\mathcal{T}_2} = \left(\frac{1}{\ell_2}\frac{d\ell_2}{d\mathcal{T}}\right)_{\mathcal{T}\geq\mathcal{T}_2} \qquad (15)$$

Substituting from (14a) and (14b) and simplifying we finally obtain:

$$\sqrt{\frac{K_\ell}{K_h}} = \frac{1+\frac{1-\alpha}{\alpha}e^{-\mathcal{T}_2}}{1+\frac{1-\alpha}{\alpha}e^{-\mathcal{T}_1}} \qquad (16)$$

which is the desired result.

In order to reproduce this result graphically, we introduce the notion of the mean time required by the reserves to respond to the call for help. Thus:

$$\overline{\mathcal{T}} = \sqrt{\mathcal{T}_1 \cdot \mathcal{T}_2} \qquad (17a)$$

with perfect intelligence, and

$$\overline{\mathcal{T}} = \sqrt{\mathcal{T}_1(\mathcal{T}_2 - \mathcal{T}_1)} \qquad (17b)$$

with no intelligence.

This characterises the average distance of reserves from the battlefield. ~~For each given~~ $\overline{\mathcal{T}}$ we calculate the values of K_l/K_h as the function of V_ℓ/V_h.

Heavy curves in Fig. 3 correspond to the assumption $\alpha = 1/3$, whereas the light curves to $\alpha = 2/3$. Continuous and broken lines correspond to the above mentioned cases of perfect intelligence, and its complete absence (call for help)."

We notice first of all that, as expected, the values of K_l/K_h needed for balance decrease with the increasing values of V_l/V_h. If the curve representing the actual relation between the effectiveness and mobility of the units runs steeper than the curves shown in Fig. 3 heavier units should be chosen in preference to heavy ones. If the actual curve runs less steep lighter units should be preferred. The curves of the Fig. 3 can be also used to select a given $\overline{T}$ (i.e. the distance of the reserves from the battlefield) which will be advantageous for any set of $\frac{V_l}{V_h}$ and $\frac{K_l}{K_h}$. A more detailed comparison with military units of various kinds will be given in the next report.

An interesting conclusion following from Fig. 3 is that, whereas in the case of perfect intelligence, equality of velocities ($V_l/V_h = 1$) leads to equality of effectiveness ($K_l/K_h = 1$), it is not at all so when the intelligence is lacking and reserves are called in only when higher losses are suffered on the battlefield. Thus, for example, we see that, for $V_l = V_h$ and $\overline{T} = 0.7$, light forces will have the edge of the battle even though their effectiveness is below 60 percent of the effectiveness of heavy forces. This apparent paradox is, of course, nothing but the consequence of the old "divide et impera" principle. Indeed, since in this case light reserves are called in immediately after the encounter of advanced units takes place, whereas the heavy reserves start only after light reserves arrived to the battlefield, the advanced heavy forces will be out numbered by lights during the middle part of the battle, and may be substantially destroyed before

the slow heavy reinforcements come in. By the time the heavy reserves arrive, they find practically nothing left to be reinforced and run into superior force of lights, thus losing the complete battle.

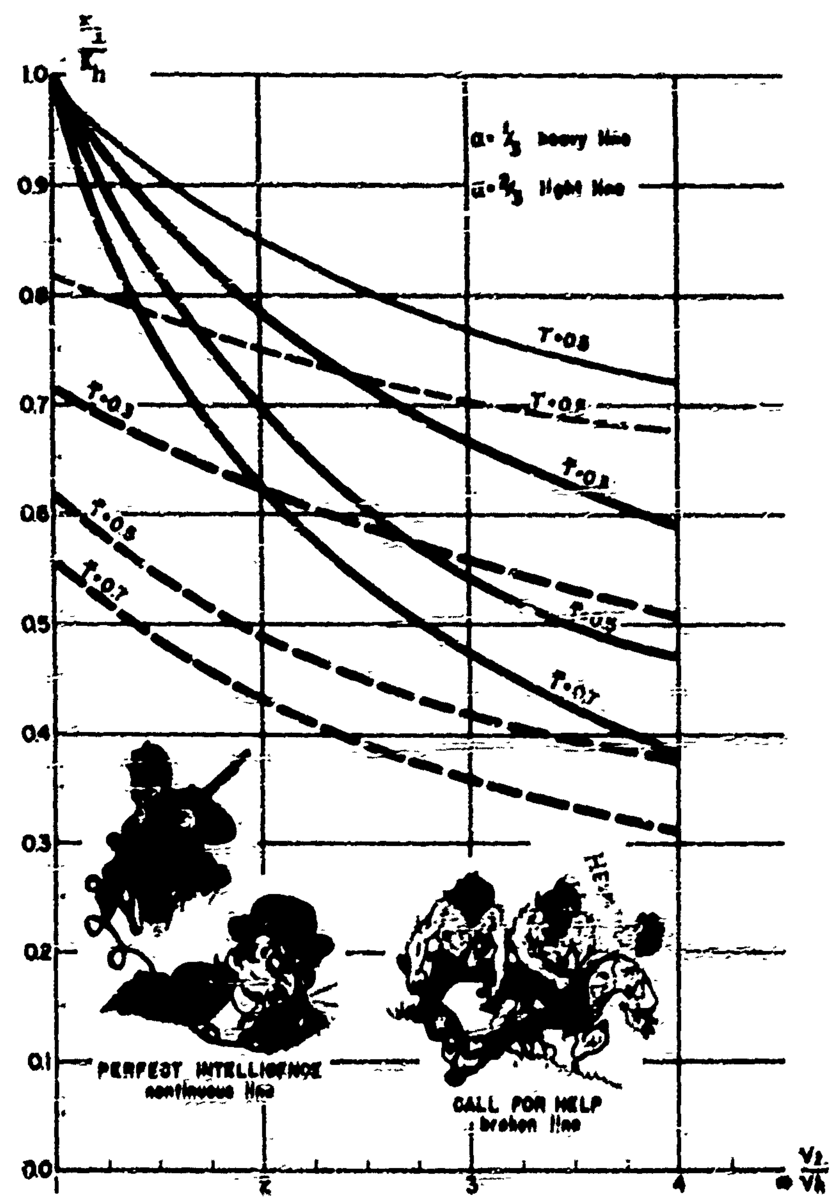
$\frac{x_i}{x_h}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
$\frac{V_i}{V_h}$
α = 1/3 heavy line
α = 2/3 light line
T = 0.3
T = 0.5
T = 0.7
PERFECT INTELLIGENCE
continuous line
CALL FOR HELP
broken line

Fig. 3

A MODEL FOR THE STUDY OF TACTICAL RESERVE FORMATIONS

by

George A. Gamow and Richard E. Zimmerman

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Dated February 1954

A MODEL FOR THE STUDY OF TACTICAL RESERVE FORMATIONS

Introduction

The conventional defensive disposition of US troops in the combat zone has been for several decades specified as "2 up and 1 back." This means that, in the Infantry Division, 2 of the 3 infantry regiments are often put on the MLR while one is held in reserve. The same can often be said about the 3 infantry battalions in each regiment, or the 3 infantry companies in each infantry battalion.

However, the threat of massed attacks by the Russians in the future, together with possible tactical changes induced by the threat or use of tactical A-weapons, suggests that a reexamination of our basic tactical concepts and formations is in order.

Indeed, both the Russians and the Germans on the Eastern Front in W W II were led to adopt defensive formations characterized by multiple lines and deployed in great depth, often to as much as 10 kilometers or more. This was evidently in response to the use, by both sides, of concentrated armor-infantry thrusts, and had no clear parallel in operations on the Western Front.

As part of a general reexamination of our current doctrine, we consider the utility of models, based on the

Lanchester equations, which can later be extended to greater detail by making use of the GEDA Analogue Computer now being acquired by COMPLAB.

Herein one intriguing calculation growing out of the above program.

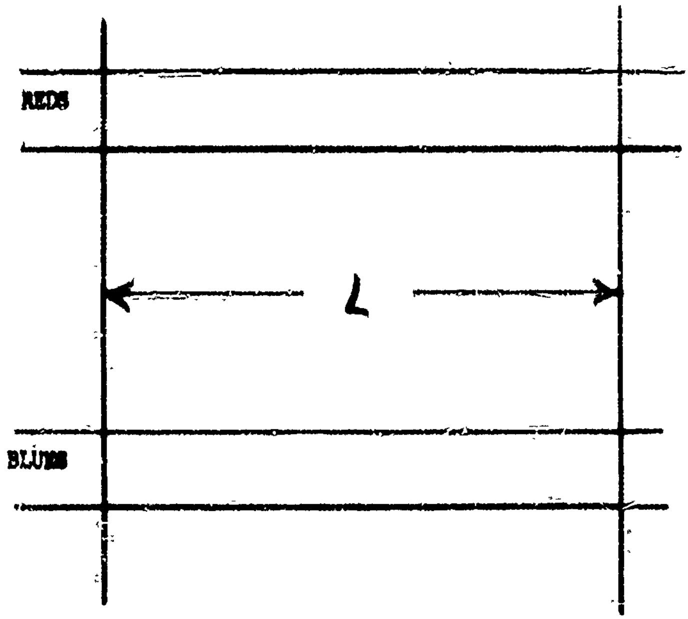
Our thanks go to Mr. E. Lee (COMPLAB) who computed the curves with the great accuracy required to find the optimum values.

Discussion:

In order to treat defensive formations in a rational manner, we construct a simple model of these formations. For purposes of this discussion, we are willing to consider that the troops on the MLR are distributed along one line, and the tactical reserves along a second line. The adequacy of this model is not considered at this point.

We imagine that all the Red forces face all the Blue forces along an infinite homogeneous front*, (see Figure 1). We further suppose that their relative strengths are so balanced that, were the Red forces to attack within any sector of front L yards without further concentration, then the resulting battle would be a draw--i.e., percentage losses on

*The results of the calculations in this memo turn out to require usually, a front of 20 miles or less. The "infinite front" is mentioned here for mathematical reasons.



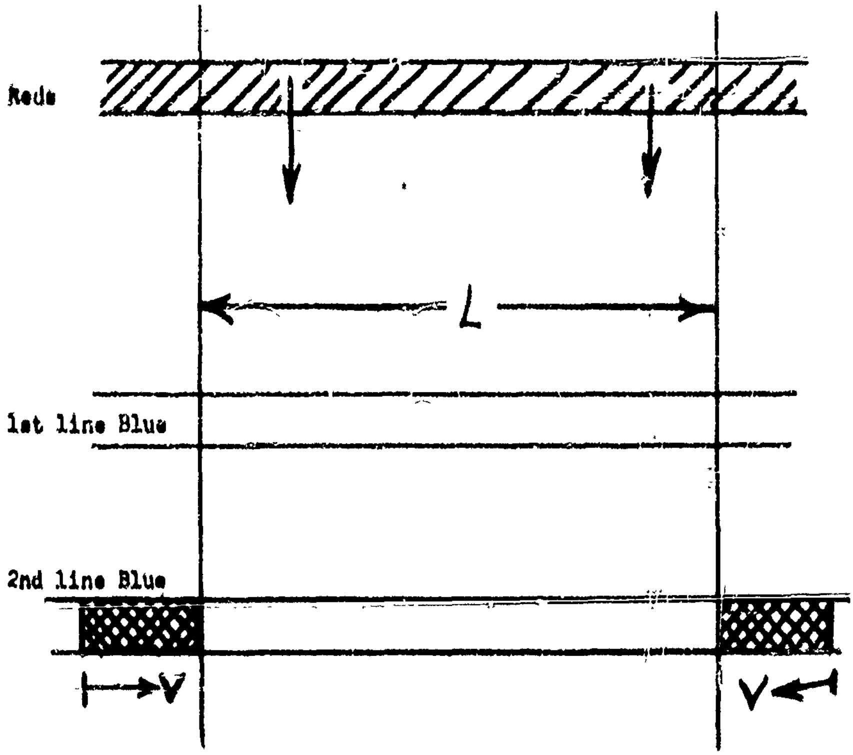
Section of front line showing Blues and Reds facing each other before the Red assault.

Figure 1

both sides are equal at any stage of the battle and at the end of the assault both sides would have been destroyed within this attack sector. Note that this must mean that the Red forces outnumber the Blues by some factor, since the defenders are in a stronger position than the attackers.

Our problem is defined by inquiring whether any advantage would accrue to the defenders if they were to split their forces into two lines, one behind the other, in some definite ratio of strengths (Figure 2). The Blues could then take advantage of the delay in the advance of the Red forces (occasioned by their battle with the first defensive line) to reinforce the second line directly behind the threatened front by drawing in troops from adjacent portions of the second line. The reinforcing troops are assumed to move with some fixed speed, V. We shall show that such a splitting of the defending force permits the defenders to destroy all the attacking force without loosing all their own men.

We take the Lanchester equations as describing the course of the battle. The mathematical derivation is given in the Appendix. Here, only the results are discussed. We will use solutions of these equations to construct graphs of the following character: we plot the number of Blue men lost at the end of the attack (expressed in units of the number of Blue men initially in the sector of width L) on the ordinate against the percentage of



Troops in cross hatched area are able to reach threatened area during time Red forces are defeating first line.

Figure 2

initial Blue strength disposed in the second line (Figure 3).

We shall get a different curve for each value of the speed V, assumed for the movement of the reserves.

For simplicity we make several additional assumptions which can be modified later:

1. The assumed equality of the Red and Blue strength (Red on offensive, Blue on defensive) will be expressed, for purposes of the mathematical calculations, as asserting each side has equal numbers of troops, each with the same killing power.

2. The battle between the attacking Reds and the first Blue line is not over until all the Blues in the first line are killed.

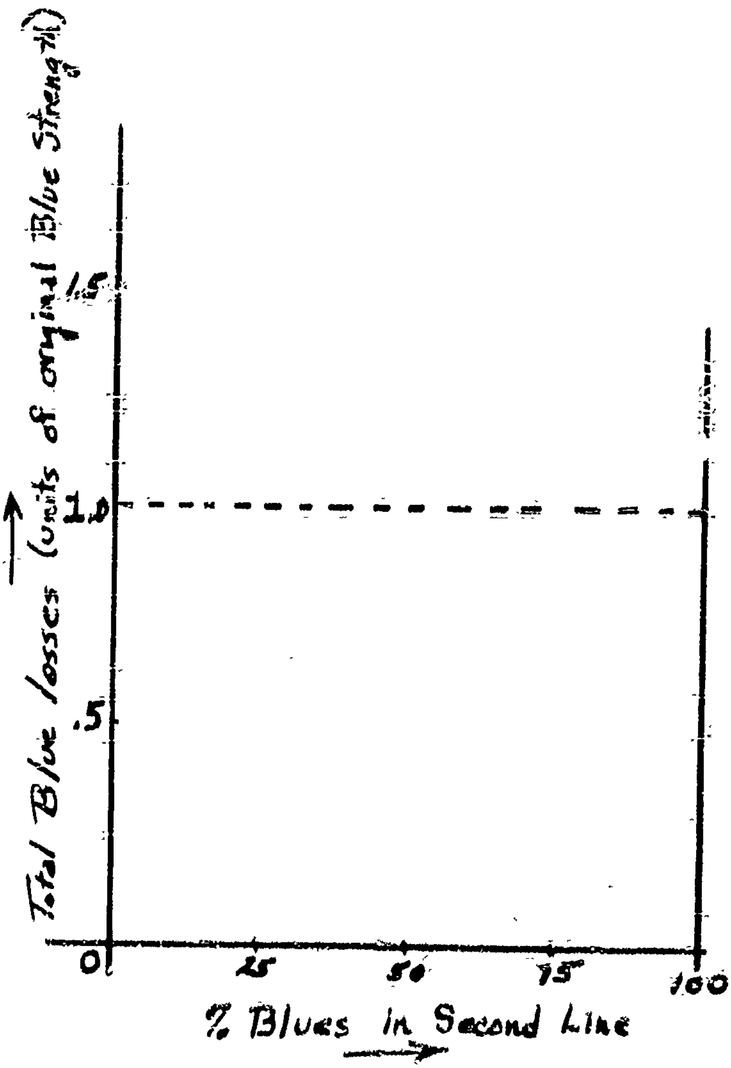
3. The time for the Reds to move from the first line after ending that battle, till they assault the second line, will be ignored.

4. Movement of Blue reserves into the second battle line stops when the second battle commences.

With these assumptions, the total number of troops lost by the Blues can be calculated for any value of the various constants and the desired graphs can be constructed. Selected curves of the results are shown in Figure 4.

While it is possible to express these results in the dimensionless form shown in the Appendix*, the curves may be made more

*See discussion following equation (14) in Appendix.



A figure showing what parameters are used in displaying the main results.

Figure 3

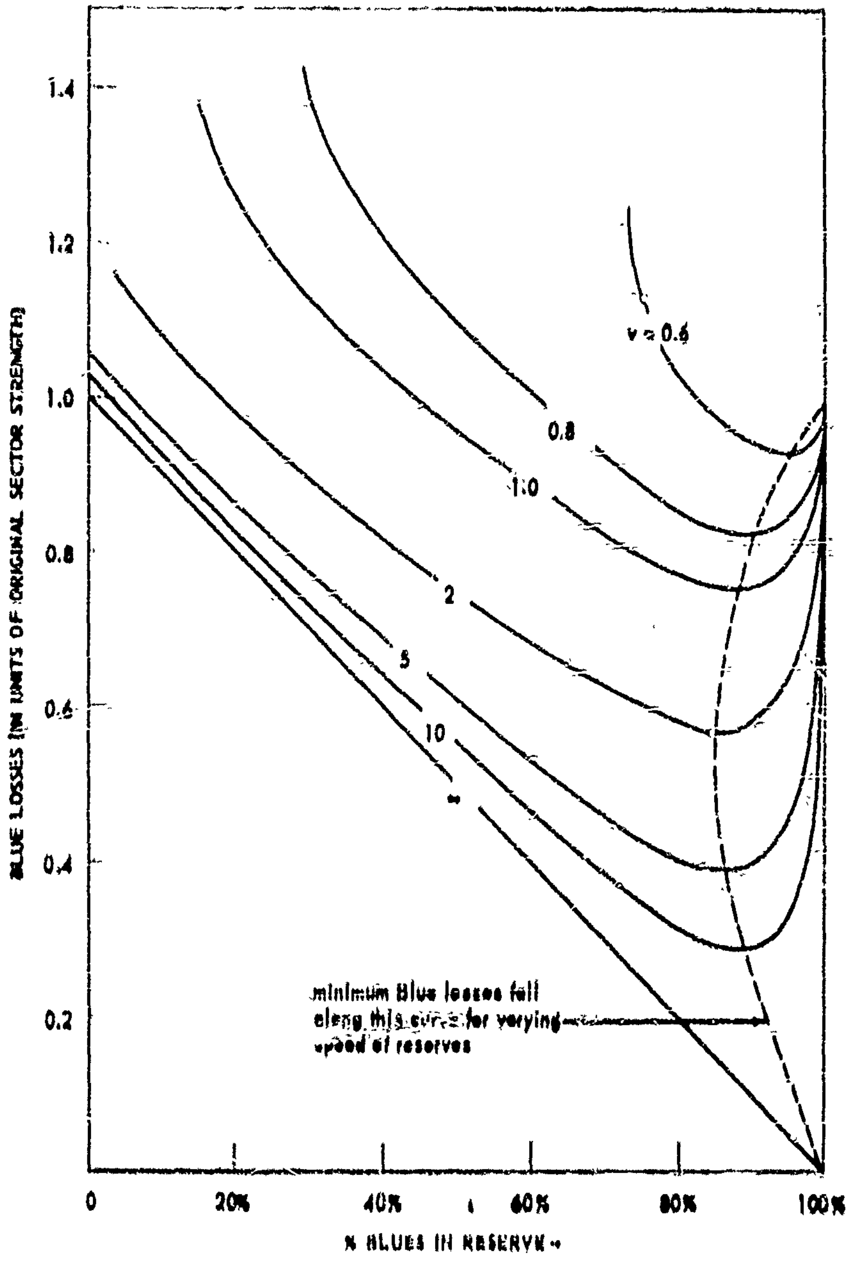


Fig. 4—Blue losses as a function of % initial strength put in reserve and for various assumed speeds of movement of reserves. Reds annihilated.

meaningful by assuming arbitrarily (a) that the assault sector is 1 mile in width, (b) initially contains 1000 Blue soldiers and 1000 Red soldiers, and (c) the killing power of each soldier in this sector is, on the average at the rate of 1 enemy soldier per hour.** Then, on the graph, $V = 1$ is interpreted as $V = 1$ mph, $V = 5$ as 5 mph, etc. Other choices for the value of the constants mentioned in (a), (b) and (c) above will simply alter the actual speed denoted by $V = 1$, $V = 5$ etc, but we will show that it will <u>not</u> change the interesting character of the results.

<u>Conclusions for this first battle:</u>

The intriguing characteristic of the results is that the optimum Blue strategy in every case favors putting the bulk of its troops (84% or more) in the reserves no matter what the speed with which its reserves may be moved. This is to be considered in terms of the doctrine of "Two up and one back".

Obviously, with such a simple model, the drawing of conclusions is dangerous. Four statements suggest themselves.

(1) The model is perhaps more a picture of the relation of the OPLR to the MLR, than of the MLR to the reserves or to a second defensive line.

(2) The disposition of US troops, conventionally deployed, must be studied more thoroughly to permit a valid comparison with the results obtained here.

**With these units, a battle between 1000 Reds and 500 Blues lasts 33 minutes.

(3) One or more essential elements are lacking in the model.

(4) Study of more detailed, realistic models should be of interest.

The case of enemy superiority

In this battle, everything will be as before except that the Reds (attackers) are assumed to have 3 times the combat effectiveness of the Blues (defenders). Note again that this may require the Reds to outnumber the Blues by a still larger factor, due to the greater combat effectiveness of the entrenched defenders, per man. The result of this set of battles is given in two different forms.

a. Figure 5 shows the minimum speed with which the reserves must move, in order that the attackers are totally destroyed along with all the defenders committed to battle, as a function of the percentage of Blues originally in reserve.

b. Figure 6 gives the total Blue loses as a function of the percentage of Blues initially in reserve for a fixed speed for the reserves of 20 miles per hour, using the same units as for the first battle where applicable. These are:

Initial Blue strength ....600 men

" Red " ......1,800 men

Killing power rate, per man, (either side).... 1 man per hour

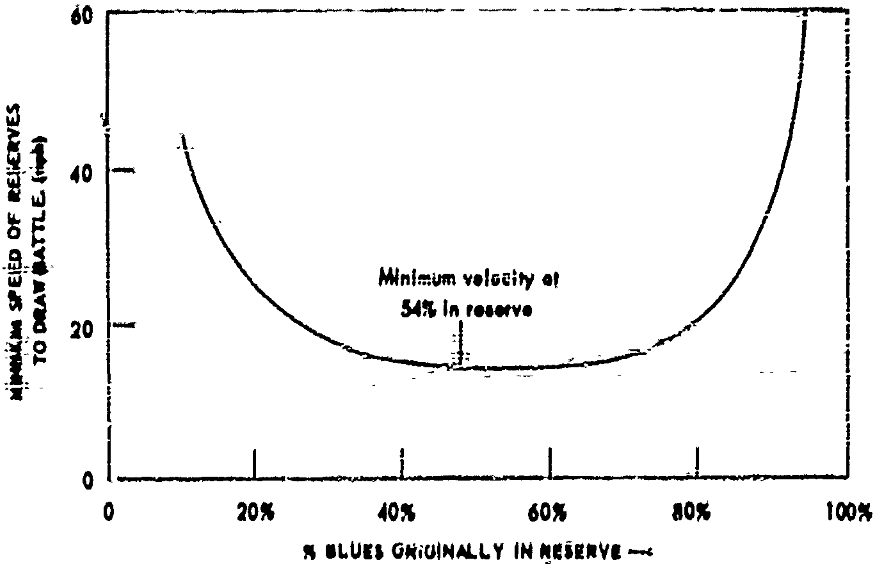


Fig. 5—Minimum speed with which reserves must move to annihilate reds, (i.e., blues are themselves annihilated).

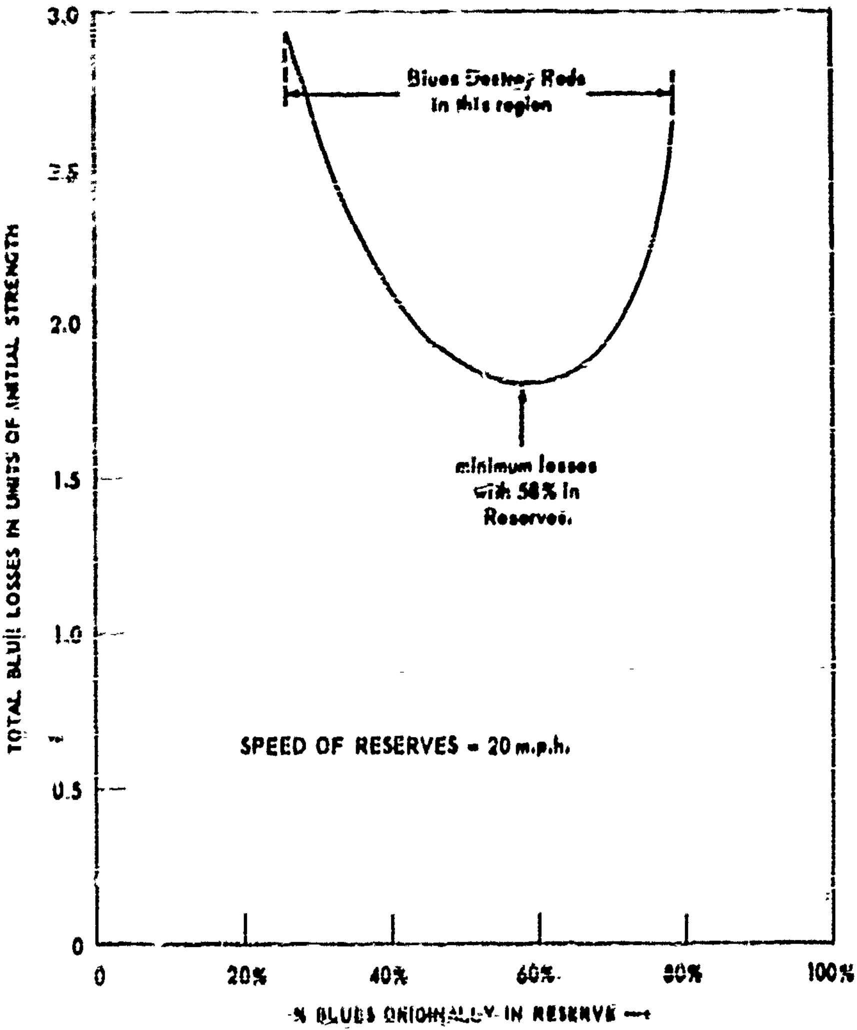


Fig. 6—Blue (defenders) losses versus percentage of Blues originally in reserve when the Reds (attackers) have three times the combat effectiveness of the Blues.

Length of front line sector.............1 mile.

Conclusions.

The first results show an optimum Blue strategy of about 54% in reserve. In this case optimum strategy is taken as favoring that percentage which yields a draw battle with the minimum speed of the reserves. As the Figure 5 shows, that minimum speed is about 14.5 in dimensionless units; or in terms of the same parameters used on page 7, 14.5 miles/hour.

This certainly indicates a limitation on the generality of the results found in the first battle (1:1 odds) where the optimum blue strategy was found to be 84% in reserve or over, for all speeds.

The results on Figure 6 show the optimum Blue strategy in the face of 3:1 odds with the speed fixed at V = 20 mph. The optimum strategy still favors about 59% in the reserves.

Thus the optimum blue strategy favors large fractions in reserve when the opposing sides are more nearly equal and less in reserve as the enemy superiority increases.

MATHEMATICAL APPENDIX

Introduction:

The Lanchester equations are a very simple formulation of the progress of a battle. They are discussed in many papers (See for example the preceding paper by George Gamow and Richard Zimmerman). They appear initially as follows:

If R = number of Red fighting units at any time

B = number of Blue fighting units at any time

k_r = number of Blue fighting units killed by each red fighting unit per unit time

k_b = number of Red fighting units killed by each Blue fighting unit per unit time

(1) Then $\frac{dR}{dt} = -k_b B$

(2) $\frac{dB}{dt} = -k_r R$

There are various forms that the solution of these equations can take. Here we derive only those required for the problem.

We note that we must answer 3 questions about each choice of the percentage of Blue troops to be put in reserve in order to determine the corresponding Blue losses.

1. How many Red troops are left after annihilating the Blue troops in the first line? (This gives the number of Reds which assault the second line, hereinafter denoted by R_{02}).

2. How long does this first battle take? (This is needed to calculate how many additional Blue troops can be moved into the threatened portion of the second line before the second battle starts. We will denote the number of blue troops starting the second battle by B_{02}).

3. How many Blue troops are left (if any) at the end of the second battle? (The difference between the number of Blue survivors, B_s, and the total number of Blues committed gives the number of Blues lost in the overall engagement and is the ordinate of the desired curves.)

To answer the 1st and 3rd questions we use the so-called "Square law." To get this we divide the two differential equations, eliminating the time.

$$(3) \quad \frac{dR}{dB} = \frac{k_b}{k_r} \frac{B}{R}$$

Separating the variables and integrating from the start of the battle to any later time gives the square law:

$$(4) \quad R_{o1}^2 - R^2 = \frac{k_r}{k_b} \left(B_{o1}^2 - B^2\right)$$

where R_{o1} = initial number of Reds in first battle.

B_{o1} = initial number of Blues in first battle.

<u>Solutions for the first battle</u>

For the first battle simplifying assumptions are that

$k_r = k_b = 1$

and $\quad B = 0$ (The Blues in the first line are annihilated)

or (5) $\boxed{R_{01}^2 - R^2 = -B_{01}^2}$

can be solved to answer the first question (and the third question if we interchange the Reds and Blues) as soon as we know the initial numbers of troops entering these battles.

To answer the second question, "How long does the first battle last?", we return to the original equations and integrate eq. (2). The general solution is:

(6) $B(t) = ae^{+Kt} + be^{-Kt}$

where $K = \sqrt{k_r k_b}$

and a, b are constants to be determined by the boundary conditions. The appropriate boundary conditions are:

(7) $B(t = o) = B_{01} = a + b$

(8) $\left.\frac{dB}{dt}\right|_{t=o} = -k_r R_{01} = aK - bK$

Setting $K = k_r = 1$ by our assumption we get from substituting (7) in (8);

$-R_{01} = a - (B_{01} - a)$

or (9) $\boxed{a = \frac{1}{2}(B_{01} - R_{01})}$

and substituting back into (7) gives

(10) $b = \frac{1}{2}(B_{01} + R_{01})$

The desired solution is then

$$(11) \quad B(t) = \tfrac{1}{2}(B_0 - R_0)e^{t} + \tfrac{1}{2}(B_0 + R_0)e^{-t}$$

We wish to solve this for the time, T,

when $\quad B = 0.$

Then, $\quad 0 = \tfrac{1}{2}(B_{01} - R_{01})e^{T} + \tfrac{1}{2}(B_{01} + R_{01})e^{-T}$

or multiplying thru by e^{T} and cancelling the $\tfrac{1}{2}$;

$$0 = (B_{01} - R_{01})e^{2T} + (B_{01} + R_{01})$$

or $\quad e^{2T} = \dfrac{-(B_{01} + R_{01})}{B_{01} - R_{01}}$

or $\quad T = \tfrac{1}{2} \ln \left(\dfrac{R_{01} + B_{01}}{R_{01} - B_{01}} \right)$

Dividing thru the numerator and denominator of the argument of the logarithm, by B_{01} gives

$$(12) \quad \boxed{T = \tfrac{1}{2} \ln \frac{r + 1}{r - 1}}$$

where $\quad r = \dfrac{R_{01}}{B_{01}}$

or $\quad r =$ the factor by which the Blues are outnumbered in the first battle.

Equation (5) and (12) are all that we need from the Lanchester equations.

To complete the solution we calculate the number, y, of reinforcing troops the Blues can bring into the threatened portion of the second line during the time, T, that the first battle lasts. Since these troops move with a speed V, we have:

y = 2 (number of troops in crosshatched area in Figure 2).

$$= 2 \left[\left\{ \text{length of area} \right\} \quad \text{times} \quad \left\{ \frac{\text{troops}}{\text{unit length}} \right\} \right]$$

$$\text{(13)} \quad \boxed{y = 2 \left[\left\{ VT \right\} \quad \left\{ \frac{p \ B_{oo}}{L} \right\} \right]}$$

where p = fraction of Blues troops put in second defensive line and all the other parameters have been defined:

V = speed of movement of reserves

T = length of first battle

B_{oo} = total number of Blue troops in the two defensive lines before the first battle.

L = length of front.

All necessary equations are now available since we can use (13) to calculate the number of Blue troops (B_{o2}) starting the second battle;

B_{o2} = reinforcements + troops initially in second line

or

$$\text{(14)} \quad \boxed{B_{o2} = y + pB_{oo}}$$

The last step is to put these equations in "dimensionless form" for ease of calculation. To do this we must choose our units of troop strength, time, and length as follows:

a. unit of troop strength will be total number of Blue troops initially present in the sector of length L.

b. unit of time will be the length of time required for one unit of either side to kill one unit of the enemy.

c. unit of length will be the length of the front, L. (Note that this choice gives V the limits of "length of front per unit kill time".) With these units the working equations are given below with all variables in the dimensionless form described above.

1. To answer first question;

from (5) $1 - R_{o2}^2 = (1-p)^2$

or (15) $$\boxed{R_{o2} = \sqrt{1-(1-p)^2}}$$

2. To answer the second question;

(16) $$\boxed{\bar{T} = \tfrac{1}{2}\ln\left(\frac{\frac{1}{1-p}+1}{\frac{1}{1-p}-1}\right) = \tfrac{1}{2}\ln\frac{2-p}{p}}$$

3. To answer the third question

(17) $$\boxed{B_s = \sqrt{R_{o2}^2 - B_{o2}^2}}$$

where R_{o2} is given by (15) above

and (18) $$\boxed{B_{o2} = (y+p)}$$

where (19) $$\boxed{y = 2\,VTp}$$

The above calculations give the total number of Blue survivors. Thus the total number lost, $\bar{C}$, is

C = (total committed) - total survivors

or (20) $\boxed{C = (1+y) - B_s}$

We include a graph (Figure 7) of equation (12) and equation (13) which is used to simplify the calculations when much accuracy is not required.

A useful approximation to equation (12) for quick calculation is given below:

$$T = \frac{1}{2} \ln \frac{r+1}{r-1}$$

Now for $r \gg 1$

$$\frac{r+1}{r-1} \approx \frac{r+2}{r} = 1 + \frac{2}{r}$$

and

$$\ln \left(1 + \frac{2}{r}\right) \approx \frac{2}{r}$$

Hence for large r:

$$T \approx \frac{1}{r}$$

Solutions for the case of enemy superiority

Solutions are required to answer the same three questions (page 14 and 15) all ready posed for the first battle. The solutions found in the preceding section are adequate for this purpose.

Thus, equation (12) will give the length of the battle with the first Blue line for any given ratio of combat effectiveness. Then equation (13) will give the number of Blue reinforcements

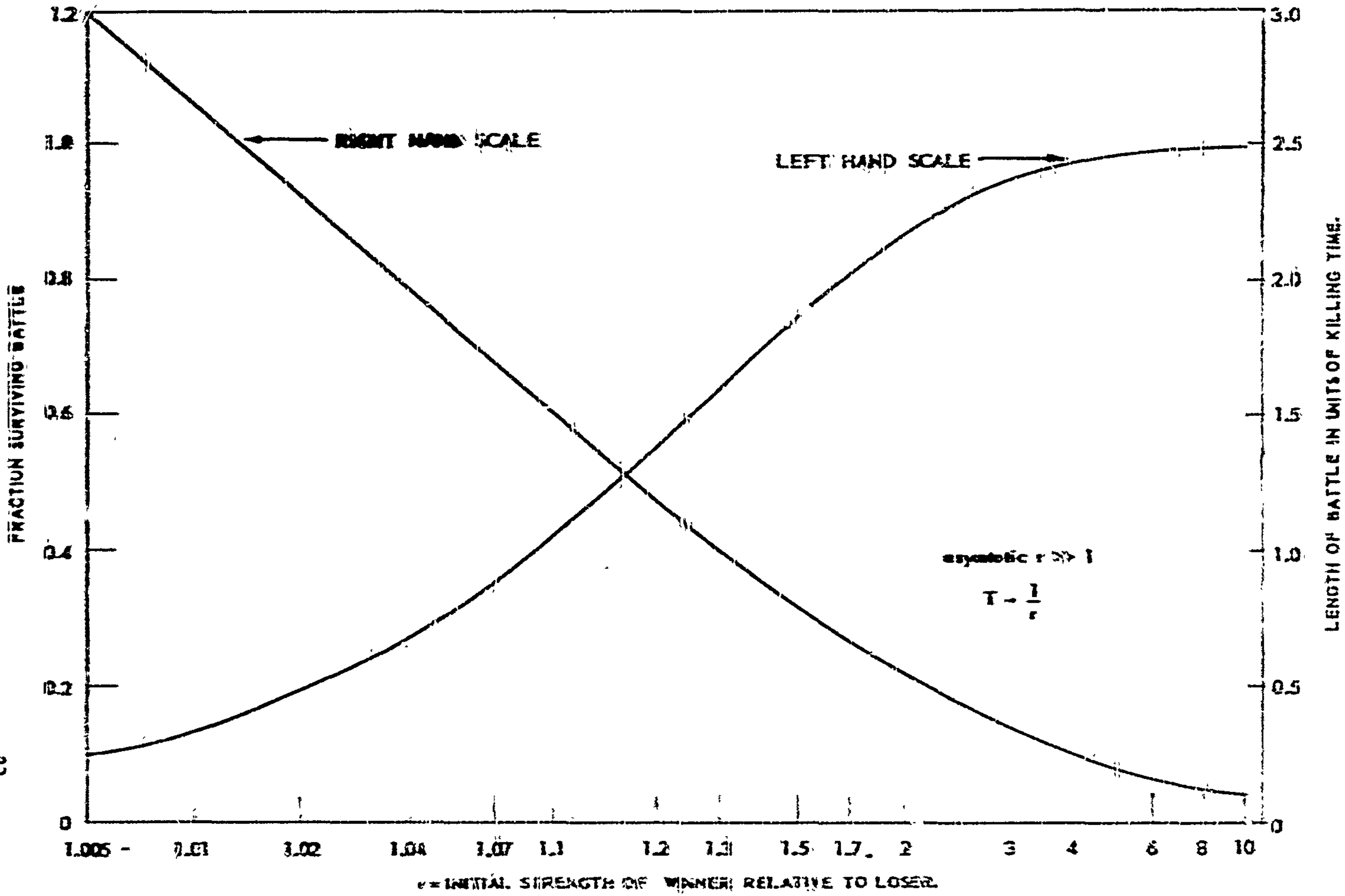

FRACTION SURVIVING BATTLE
LENGTH OF BATTLE IN UNITS OF KILLING TIME.
RIGHT HAND SCALE
LEFT HAND SCALE
asymtotic r >> 1
$T \to \frac{1}{r}$
1.2
0.8
0.6
0.4
0.2
0
3.0
2.5
2.0
1.5
1.0
0.5
0
1.005
1.02
1.04
1.07
1.1
1.2
1.5
1.7
2
3
4
6
8
10
r = INITIAL STRENGTH OF WINNER RELATIVE TO LOSER

Figure 7

which can move into the threatened portion of the second Blue line, while the battle at the first Blue line progresses. Equation (14) gives the number of Blue forces which start the battle for the second line; while the square law [eq. (5)] will give the number of Red survivors from the battle of the first line. Finally, use of the square law again will give the survivors of the second battle. These results can then be tabulated or used to make the graphs presented.

MONTE CARLO METHOD IN WAR-GAME THEORY

by

George A. Gamow

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Dated 1 April 1952

MONTE CARLO METHOD IN WAR-GAME THEORY

The purpose of this report is to discuss the possibility of constructing simplified models of various tactical situations which may arise in various types of military engagements, and of studying these models by statistical method based on averaging over a large number of games played according to fixed rules but with random selection of individual moves. The well-known "Kriegspiel" in chess can be considered as one kind of such a model, although the rules pertaining to chess-figures do not seem to correspond to any actual military entities, not even probably to the implements of war dating back to the time of the ancient world. Thus, as a working example, we will select a simple model corresponding to an engagement between two tank forces in a partially wooded flat country.

The battlefield will be represented by a lattice of hexagonal fields which are selected because they correspond to a higher degree of isotropy than the regular square lattice of the chessboard. A certain fraction of hexagons is cross-hatched to represent the wooded areas, whereas the others are white corresponding to the open fields. It goes without saying that the regular check pattern is not maintained, so that the cross-hatched hexagons can be clustered into groups representing wooded areas of different dimensions. If no topographical advantage is to be given to either tank force, the pattern must be more or less symmetrical in respect to the central line. One such simplified battlefield, actually constructed in ORO, is shown in Figure 1.

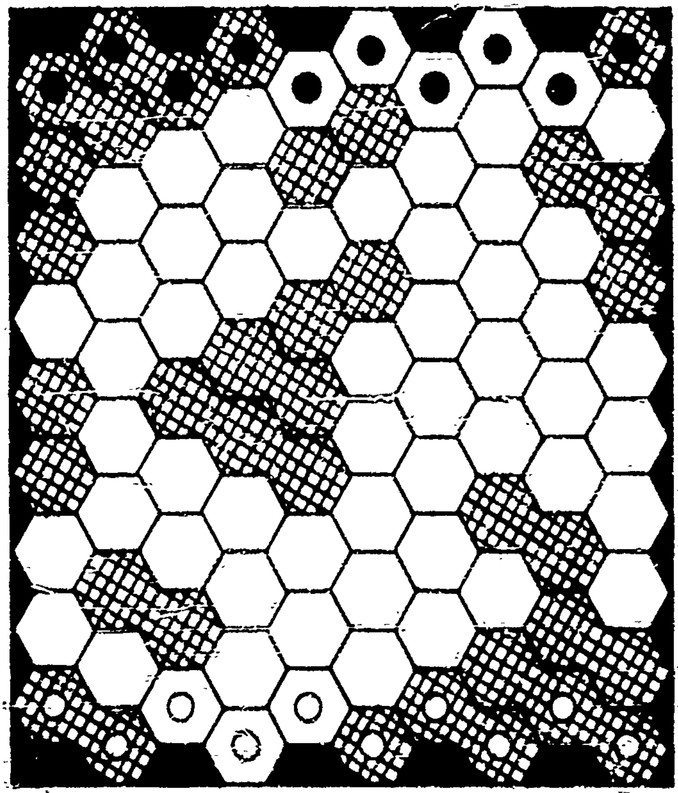

Figure 1

The two opposing tank forces, ten units each, are originally located at the rear lines of the battlefield, and "a move" on each side consists in displacement of each of the tanks to one of the adjoining hexagonal fields (although not all tanks must necessarily be moved).

If two opposing tanks come to adjoining white fields, "a battle" is announced, and its outcome is decided by tossing a coin or a die.

If, as may happen, a moving tank comes in contact with two enemy tanks simultaneously, it must "shoot it out" first with one of the tanks, and, if victorious, with the other. (More realistic rules can also be introduced in that case.)

If a tank on a white field is in contact with an enemy tank in a cross-hatched field (considered as concealed), the first tank is always killed (or given a much higher probability of being killed in the dice-tossing process). If both tanks are in the woods, a battle is announced only if one of them moves into the field occupied by the other (half see distance), and the outcome is again decided by a die.

The objective of the game may be: the destruction of a maximum number of enemy tanks with least losses for oneself; the destruction of some objective located at the rear-line of enemy forces; or still some other aim.

If the game is played by hand by two individuals, they must be seated (just as in classical Krieg-Spiel) back to back, facing their own copy of the battlefield showing only the position of their own tanks. The battles, and their outcome is announced by an umpire who observes both boards. Several such games were played in ORO, and turned out to be quite amusing.

The main purpose of the game is, however, a random playing in which the motion of each tank of both forces is decided by tossing a die. While in "intelligent" playing different strategies will be used by players themselves, the strategy in a random play must be "inbuilt" into the rules of random tank movement. Thus, for example,

the strategy of massing all tanks together or dispersing them all over the field may be introduced by bringing in "attractive" or "repulsive" forces between the tanks of the same tank force. This would require a modification of a simple dice-tossing technique to the extent that the tanks will have preferential probability of moving towards, or away from the center of gravity of other tanks of the same force.

Playing such a random game by hand would certainly be a very lengthy and boresome occupation, but the entire idea is that games should be played by an electronic computing machine which can probably accomplish a few hundred complete games per hour. Among a sufficiently large sample of such games played under the same rules and the same initial condition, there will be a large percentage of even exchange, but there will also be the games in which one or the other side achieves a decisive victory. If one plots the results, selecting as a characteristic the number of victorious tanks left after all enemy tanks are destroyed, one can expect a curve of the type shown by the solid curve in Figure 2. The curve will be, of course, symmetrical in respect to the center if both sides have exactly equal advantages and disadvantages.

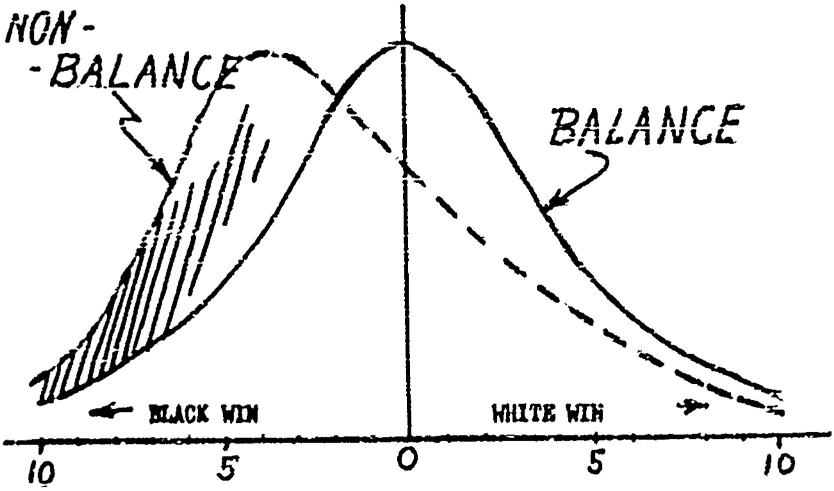

Figure 2.

Suppose now we increase the speed of black tanks by, say, 10 percent (giving black forces 11 moves per each 10 moves of the white forces), and at the same time decrease their armor or fire power by giving them, say, 45 against 55 chance in the battle. If the increase in speed exactly balances the increase in vulnerability, the curve of Figure 2 will remain symmetrical. If the advantages and disadvantages are not balanced up, the curve will become screwed, showing which choice should be made. Thus the above method should be able to give a direct quantitative answer on the relative value of various improvements.

In the same way one can test out various strategies by giving, for example, a certain degree of clustering tendency to the black tanks and of dispersing tendency to the white. Or else one can test the relative advantages of higher speed connected with higher vulnerability, in its dependence on the type of strategy adopted by a given tank force.

It goes without saying that the particular example discussed above should be considered only as the first step in the development of the Monte Carlo method in the study of tactical situations. If, as is expected, the study of that simple example shows promising results, the next step would be the application of similar methods to other possible military situations, and the formulation of more realistic rules of the game. Of crucial important here will be, of course, the selection of "models" most appropriate for various types of actually existing situations, and of the set of "rules" for the game which, being sufficiently realistic, would, however, not overcharge the abilities of existing electronic computers.

THE APPLICATION OF ELECTRONIC COMPUTERS TO
MONTE CARLO WAR GAME PROBLEMS

by

Richard E. Zimmerman

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Dated 24 November 1952

THE APPLICATION OF ELECTRONIC COMPUTERS TO

MONTE CARLO WAR GAME PROBLEMS

Introduction

The tentative project "Tin Soldier" "sponsored" by George Gamow[1] is investigating the application of electronic computers to be problems of "Monte Carlo War Games." Tin Soldier is working closely with the Computation Laboratory and also with Project ARMOR since the first group of problems to be attacked concerns tank battles. It is contemplated, however, that the same methods will be applied for the solution of problems for all possible kinds of military units. Certain men from the Los Alamos Scientific Laboratory have participated in the initiation of this work[2] partly due to their access to a MANIAC Digital Computer and the IBM CPC II. While no complete problems have been run on these machines due to the pressure of other work, enough work was done to convince Dr. George Gamow and the Los Alamos Laboratory men that the idea showed promise.

Tin Soldier hopes to provide a technique for the detailed analysis of military tactical doctrines and weapon design which approaches much more closely the military realism of an actual battle (or at least practice maneuvers) than does the simple mathematical analysis of the past. It proposes to do this by using the machine capacity for massive and tedious calculation to "play" a kind of military chess game or "Kriegspiel" but including

1. See preceding paper.
2. See Appendix A for a description of this work.

a very detailed description of the tactics and weapons designs.

We describe the general nature of the Tin Soldier approach. Imagine that a contour map is marked to indicate the pre-battle positions of two opposing tank units. Call them the Blue and Red sides respectively. Suppose that one man directs the Blue side and another directs the Red through an imaginary battle. One way to organize the progress of this battle would be for the two men to agree to a set of rules like the following:

> (1) Each man in his turn is permitted to move his tanks a distance which would correspond to 10 seconds elapsed time on the real terrain, consistent with the capabilities of the tank.
>
> (2) After each man moves his tanks according to his military good sense, tanks within range are assumed to bring fire on the opposition. The winner of tank duels is decided by flipping a loaded coin which is supposed to express the odds on the battle outcome in actual fighting between these tanks. For example, if the Blue tanks are M46's armed with 90 mm guns and the Red tanks are the German Mk III's armed with a 50 mm gun, then it might make military sense to give the M46 four-to-one odds over the Mk III. The loaded coin should therefore name the M46 the winner 80% of the time and the Mk III 20% of the time.
>
> (3) The battle is over as soon as all the tanks on either side have either been knocked out or have successfully withdrawn from the battlefield.

The above set of rules merely describes a kind of map exercise long used for the training of officers. We note that rule (1) is so phrased that the men playing the game are required to be familiar with the use of tanks. They exercise their military judgement at every step of the game. Contrariwise, rules (2) and (3) are quite simple and do not require trained men to apply them. They do however require trained men for their formulation. The crux of the Tin Soldier approach is to have trained men replace the vague

rule (1) by a set of very specific rules which layman can follow. If this can be done and still make some military sense then the rules can be put directly into mathematical form and the entire "battle" run out on desk calculators by semi-skilled technicians. Or the battle can be fought many hundreds of times more quickly on an electronic computer.

It is Tin Soldier's contention that this can be done well enough to make military sense.

The above set of rules also brought into the picture, in a very natural way, a probability function. That is, the outcome of a tank duel was not specified as a certainty but rather as a weighted probability. In this fashion an exceedingly lengthy account of all the detailed factors which actually ~~determine who~~ wins in a M-46-MkIII duel was avoided. Tin Soldier hopes to be able to make extensive use of such simple probability functions in setting up the rules controlling the battle. This would serve two purposes; (1) it is probably a necessary step in reducing the complexity of the battle rules to the point where they are tractable for machine calculations; (2) they will help to indicate how sensitive the outcome of a specified battle would be to the confusion always present to some degree in actual battle.

The range of problems which Tin Soldier should be able to investigate coincides with the degree of detail included in the rules of the game. Thus to the extent that there are detailed tactical rules governing the progress of the battle, modifications of these tactical rules will demonstrate their strength and weakness by their influence on the outcome of the battle. All

tactical problems, all questions of weapon design, whether following conventional or unconventional lines are potentially proper subjects for investigation by this approach. The limitation is simply imposed by the one factor: how complicated a battle the machine can process in a reasonable time.

A Test Problem.

Conversations just completed by an ad hoc committee under the direction of Dr. Nicholas Smith and including both ORO and Los Alamos Laboratory personnel have resulted in the formulation of a simple tank breakthrough problem with some intrinsic merit.[(1)] However this problem is not expected to supply meaningful military answers about tank design or tactics but is to be used to explore in more detail the techniques of reducing a description of the rules by which the tank battle evolves to the simple mathematical language amenable to machine calculations.

For the sake of concreteness, the test problem is interpreted as supplying an answer to the question; "In attacking a defensive line of tanks possessing a mobile reserve, is it better for the attackers to invest a fixed sum of money in many light tanks or fewer heavy tanks?" A linear relation is assumed between the weight of a tank and its cost; between the weight of a tank and the weight of its armor; and between the weight of a tank and the thickness of the armor which the gun it carries can penetrate. Conventional tanks have been found to obey very roughly

(1) Dr. J. Harrison is now converting the general rules of the game into the detailed form necessary for coding the problems for a computer.

these relations.[1] Current American tank parameters will be used to fix the value of any constants required in the above relations. The cost in dollars of all the attacking tanks is to be equal to the cost of all the defenders.

The rules of the game, taken together, specify the physical capabilities of the tanks and the tactics they employ in the battle. The general character of the rules adopted by the committee is as follows:

(1) There shall be 10 defending tanks, part of them committed to hold a defensive line, and restricted in their maneuvers to motion along the line. The defensive tanks not committed to the line will form a mobile reserve. They will reinforce any threatened sections of the defensive line.

(2) The attackers will be variable in number according to the restrictions on cost, weight, armor thickness and gun already described. Their mission will be to advance through the defenders and reach a "goal line" behind the defending tanks. The game will be over when some specified number of the attackers (probably all survivors) reach this goal line, or when all the tanks on either side are destroyed. The tactics of the attackers will be to probe the defensive line for a weak spot so as to avoid a frontal assault on the defensive line if possible.

The committee further accepted the following recommendations for directing these initial exploratory calculations:

(a) The complete game should be restricted so as to take the order of one minute of running time on the MANIAC (or ORDVAC) machine.

(b) Calculations should be carried out for at least two different terrains.

(1) According to a private communication with Mr. E. Dunn, of the British ORG, temporarily with ORO. There are, of course, many variations from this, generally in the direction of lighter guns than the rule suggests.

(c) A total of 10 to 100 games should be played.

(d) The tactics, or rules of motion of the tanks, should be varied over a few of the games at the start to insure that there will actually be a contest between the opposing tanks and to determine how sensitive the outcome of the game is to moderate variations in these tactics.

(e) The exploratory program should include at least a few games coded for or actually run on machines other than the MANIAC, e.g. the UNIVAC, Defense Calculator and FERRANTI style. Further the coding of this problem for the proposed Bell Telephone Laboratory rotating drum computer should be investigated.

(f) Various approximations for the calculation of the distance from one tank to another may be used in the interest of simplicity and saving of time. The exact calculation of $r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ involves two multiplications. Here x_1, y_1 are the coordinates of one tank and x_2, y_2 are the coordinates of the second tank. On the MANIAC, the time for one multiplication is about 1.5 milli-seconds (0.0015 sec). A single addition (or subtraction) takes only about 1/40 of this or about 40 micro seconds. Hence the interest in reducing the number of multiplications required in the course of the game.

(g) The program described above is expected to require about 1 man-year.

Machine Capacity

Before setting up a program for Tin Soldier we must deduce the limits imposed on such a program by a computer's capacity for calculation. Thus we must consider the general nature of the problems Tin Soldier proposes to study.

To this end we now suppose that we have found a set of rules which define a militarily realistic tank battle. Suppose further that it is desired to determine the optimum compromise between the amount of armor that a tank of specified total weight should carry as opposed to the size (weight) of the gun. This compromise can only be expected to be optimized for a particular style of battle and for action against a particular style of enemy tank.

The first step in the calculation is to assign a value to the weight of the armor and the weight of the gun. Then a sufficient number of games must be played with this particular choice of weights to establish the distribution of win-lose probabilities. If a hundred games were played the result might be as shown by the histogram in Fig. 1.

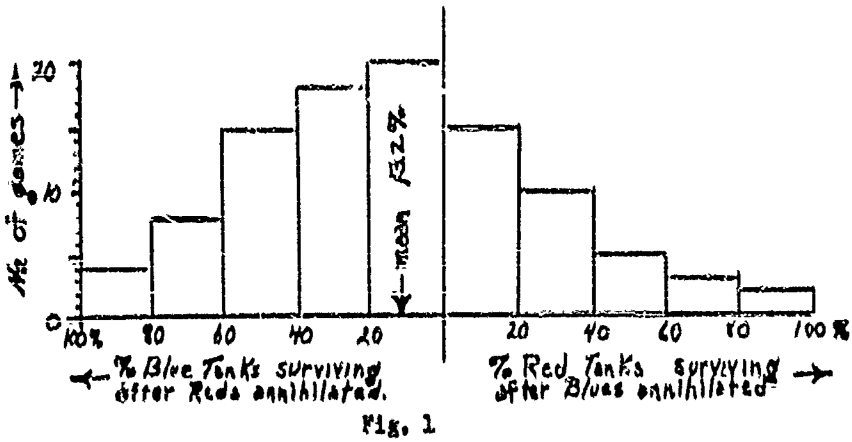


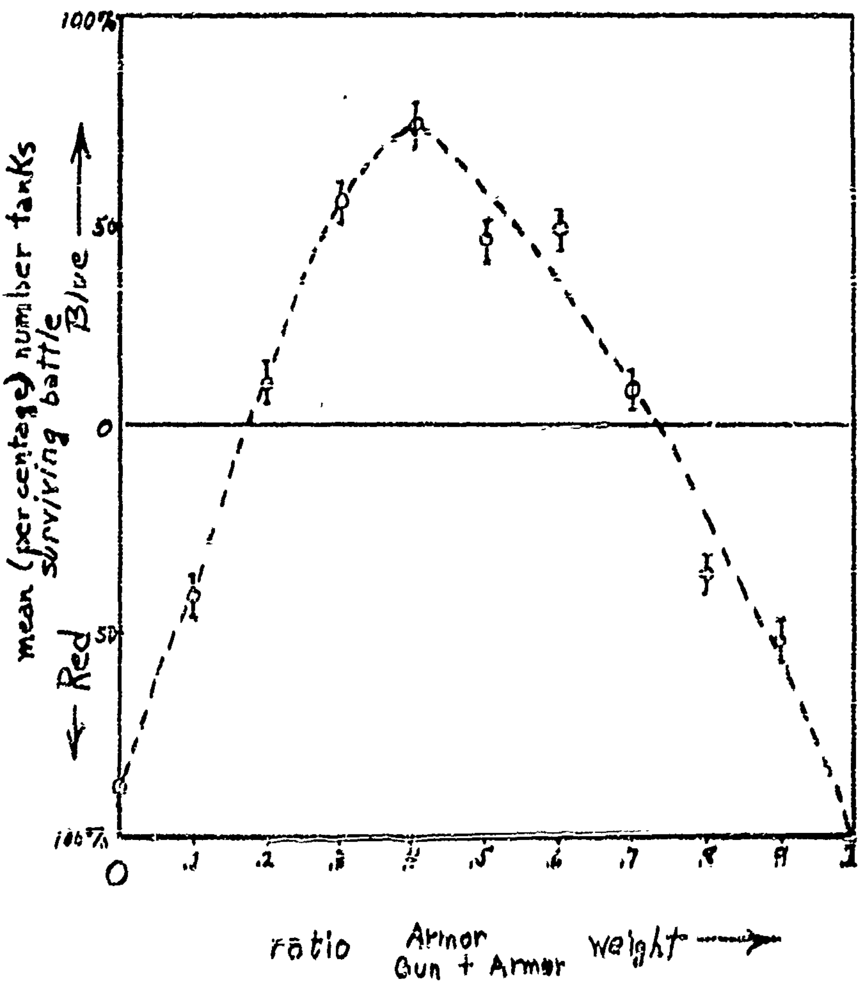
Fig. 1

This histogram shows that the Blues have the edge in the battle; they won 65% of the battles and lost 35%. The (arithmetic) mean outcome was a Blue victory with 13.2% of its tanks surviving. The standard deviation of this mean, that is the interval inside which we are 67% confident that the true mean lies, is 13.2 ± 4% (approx.)

One must note that as the dispersion (broadness) of the win-lose frequency cure decreases, so does the number of games required to establish specified confidence limits of the mean. Thus one cannot tell beforehand how many games must be played to establish the mean battle results. It is felt however that if the frequency curve is so broad that 100 games will not be sufficient to fix the mean to within about 5%, then the results are probably not believable anyway. In other words, one would probably require a rather clear cut indication of a design superiority from such problems. This would provide a margin of safety against being mislead by the simplifying assumptions used in formulating the rules.

To continue this hypothetical problem, our purpose was to optimize the armor-gun compromise in the Blue tanks. The 100 games played above gave us the mean battle ability of the Blues for <u>one</u> such compromise.

We must now choose another value of this weight ratio and play another set of (about) 100 games. For each set of 100 games we will get one point for the construction of the following curves (Fig. 2) See page 9.



100%
50
0
50
100%
mean (percentage) number tanks surviving battle
← Red
Blue →
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
ratio $\frac{\text{Armor}}{\text{Gun + Armor}}$ weight →

Fig. 2

The curve in Figure 2 is very well defined by 10 points; thus about 1000 games are required to develope it. The result of these hypothetical calculations then is that the optimum $\left(\frac{\text{Armor Weight}}{\text{Armor + Gun Weight}}\right)$ ratio is about .4 for the game with this particular set of tactical rules, for this particular enemy tank design, for this particular terrain, and for the particular weapon systems involved in the battle.

Only to the extent that we believe the game has mocked up a real military situation do we also believe that .4 is the best $\left(\frac{\text{Armor weight}}{\text{Gun + Armor}}\right)$ ratio for a tank operating in that military situation.

In order to develop confidence in the above result it will probably be necessary to determine how sensitive the final result is to moderate changes in the rules of the game. We might have to reject the answer if, for example, only slight changes in terrain or in the tactics would radically alter the optimum $\frac{\text{Armor}}{\text{Armor + Gun}}$ weight ratio. Thus we are obliged to repeat the entire process (involving around 1000 games) for various altered but similar games. Perhaps 10 such repetitions would be the order required.

Thus the total number of games required to make a fairly detailed analysis of the ARMOR-GUN weight ratio under specified circumstances may be on the order of 10^4 games.

If each game took, on the average, one minute of computer time, then this represents about 1 full months work on the MANIAC, ORDVAC, UNIVAC or Defense Calculator.

It should now be clear why the time problem is a serious one. There are at least 5 lines of attack on the time limitations.

(1) Intensive investigation of coding short cuts and approximations to replace time consuming exact calculations.

(2) Use of analytical calculations where possible to replace sub-routines involving stochastic variables.

(3) Reduction in the generality (i.e the number of probability parameters) of a game with consequent decrease in the number of games required to fix the mean battle outcome.

(4) Make maximum use of small unit game results before considering large unit actions.

(5) Press the investigation of the use of symbolic logic which lends itself more easily to coding for the Bell Telephone rotating drum machine. If successful, this approach could reduce the time required to play one game by a factor of perhaps 1000. In such a case, 10^4 games could be played in a matter of minutes or hours, instead of the month suggested above.

INITIAL CONDITIONS

In the preceding section an outline was given of the manner in which a computer might be used to optimize the division of weight between the armor of a tank and the gun it carries. In order to carry out these calculations it is necessary to establish a definite relation between the weight of a gun and its penetrating power.

Clearly there is no one-to-one correspondence between these two factors, i.e., a 500 pound weapon might be a high velocity 76 mm gun firing an armor piercing shell, or a low velocity 105 mm howitzer firing HEAT rounds (shape charges), or a rocket launcher firing 1000 pound projectiles. Such weapons, even though their weight may be the same, would differ widely not only in armor penetration at some fixed range, but also, in many other characteristics such as accuracy, time between major maintenance of the weapon, and the number of rounds of ammunition which can be carried. Thus before the calculation described above could be started it would be necessary to decide what kind of weapon would be considered and to fashion a realistic measure of the penetrating power of the weapon versus its weight. Any significant improvement in the design of armor penetrating weapons would render the entire calculation obsolete and it would have to be repeated using the new gun parameters. This emphasizes again that a computer playing the Tin Soldier game is only an analytical tool for use with data and assumptions derived by some other process.

CHECKING PROCEDURES

Here as elsewhere in military research and development one has no insurance against mistakes of interpretation. Only a full scale war can provide a completely valid test of military doctrines. Precisely the same measures must be taken with any results of Tin Soldier as are taken with any other analytical results. One can check the results against historical battles; one can introduce the results into maneuvers of troops to test their efficacy.

ENLARGING THE SCOPE OF THE PROGRAM

Formally it is clear that the rules of a game need not be interpreted as describing a battle involving tanks. Thus if we assign one of the maneuvering elements an operational speed of 2 miles an hour, give its weight as 200 pounds, supply no armor and a weapon able to penetrate 3/8 inch of armor at a range of 100 yards together with the ability to cross every kind of terrain, then we make sense if we call this maneuvering element an infantry man or squad. Thus to a reasonable approximation the problem can be made to include entire weapon systems by merely altering the numbers associated with the firepower, mobility and vulnerability of a unit.

Of course the detailed tactics employed will vary from one weapon to another, but within limits this can be taken care of by altering the probability parameters which control the motion of the unit and degree of interaction with other units. Thus the call for help, the enveloping maneuver, a probing for a weak section of the front, the concentration of weapons, and setting up a stationary strong point are factors common to varying degrees in the employment of all weapons.

In principle then, the problem can be enlarged without introducing any new difficulties except for the increased time required to play the game. Any desired complexity in the terrain, the tactics or the weapon systems can be put into the game in a simple, direct but time consuming manner. If the approach adopted

by Tin Soldier will work for small unit actions, then the only limit now expected on its expansion to any degree whatsoever would appear to be the time limitation already described.

SUMMARY:

1. A brief feasability study on the application of computers to Project ARMOR has been completed with favorable recommendations;
2. A period of exploratory work has now started to develop techniques of breaking down military tank problems for machine processing.
3. The main effort will then follow. That is, it must be assured that sufficient military realism is in the problem to justify accepting the results of the calculations.
4. The limitations on how far the program can be carried derive mainly from the restriction on the length of time one problem takes on the machine. These limitations should be attacked in two ways:

   (a) refinement of the present approach;

   (b) development of one-step logical operations[1] to pass from one battlefield state to the next in the order of 10 to 100 micro seconds calculation time of the computer.

1. Dr. E. Cushen of the Computer Lab is considering this problem.

APPENDIX "A"

THE CODING OF A SIMPLE BREAKTHROUGH PROBLEM

The LASL men initially considered a very simple tank breakthrough problem in order to make a feasibility study of the Tin Soldier approach. Its treatment is presented here. The problem as described has virtually no military merit and is so simple that hand calculations would be sufficient to carry it through. Nevertheless, parts of the problem were coded for the IBM CPC II and enough runs were made to satisfy us that the general procedure was workable. In addition the problem was described to personnel working with the Los Alamos MANIAC and they quickly turned out a so-called "flow diagram". This diagram is shown in Figure 3. On it every calculation required for playing the game is completely specified in a form understandable to any person familiar with large scale computers. The symbolism follows that of J. von Neuman and is fairly standardized over the country. Using the flow diagram the control orders for a computer may be written down immediately.

The problem involves a situation described most compactly by the block diagram in Figure 4. The action is described below.

A line of anti-tank guns, deployed in depth and with no overlapping fields of fire, is approached by a column of 10 tanks. The motion of the lead tank in this column is strongly weighted in a forward direction but varies somewhat in a random fashion. The tank column "follows the leader", i.e. the $(n + 1)$th tank always moves into the position just vacated by the n^{th} tank.

As soon as a tank comes within range of an anti-tank gun, the anti-tank gun gets one shot at the tank. A selection among random numbers then decides whether the tank was killed. If the tank was not killed it moves again after which the anti-tank gun gets another shot at it. The anti-tank gun gets only one shot between each move and always shoots at the lead tank in that part of the tank column within range. If the anti-tank gun misses its second shot, the tanks move again after which the anti-tank gun shoots a third time. Immediately after this third shot all surviving tanks within range of the anti-tank gun get one shot at the anti-tank gun. So long as the anti-tank gun survives, the tanks continue alternating a move with one shot from the anti-tank gun followed by one shot from each of the surviving tanks. The odds are generally against the anti-tank gun and it is eventually knocked out. The column then continues its motion forward until it comes within range of a second anti-tank gun where the move-fire-move-fire routine is repeated. Finally, the head of the tank column emerges from the line of anti-tank guns. The score of the game is the number of tanks lost in breaking through.

The parameters adopted for hand calculations and for running on the IBM CPC II were that each move corresponded to a 10 second interval on the battlefield. In this time the tank generally moved about 100 yards which corresponds to a speed of about 20 mph. Specifically, for each move a digit was selected at random from a table of logarithms. If the number was between 0 and 5 inclusive,

the tank advanced 100 yards forward. If the number was 6, the tank moved 50 yards to the left; if 7 then 50 yards right; if 8 then 50 yards to the rear and it stayed in position if the number was 9.

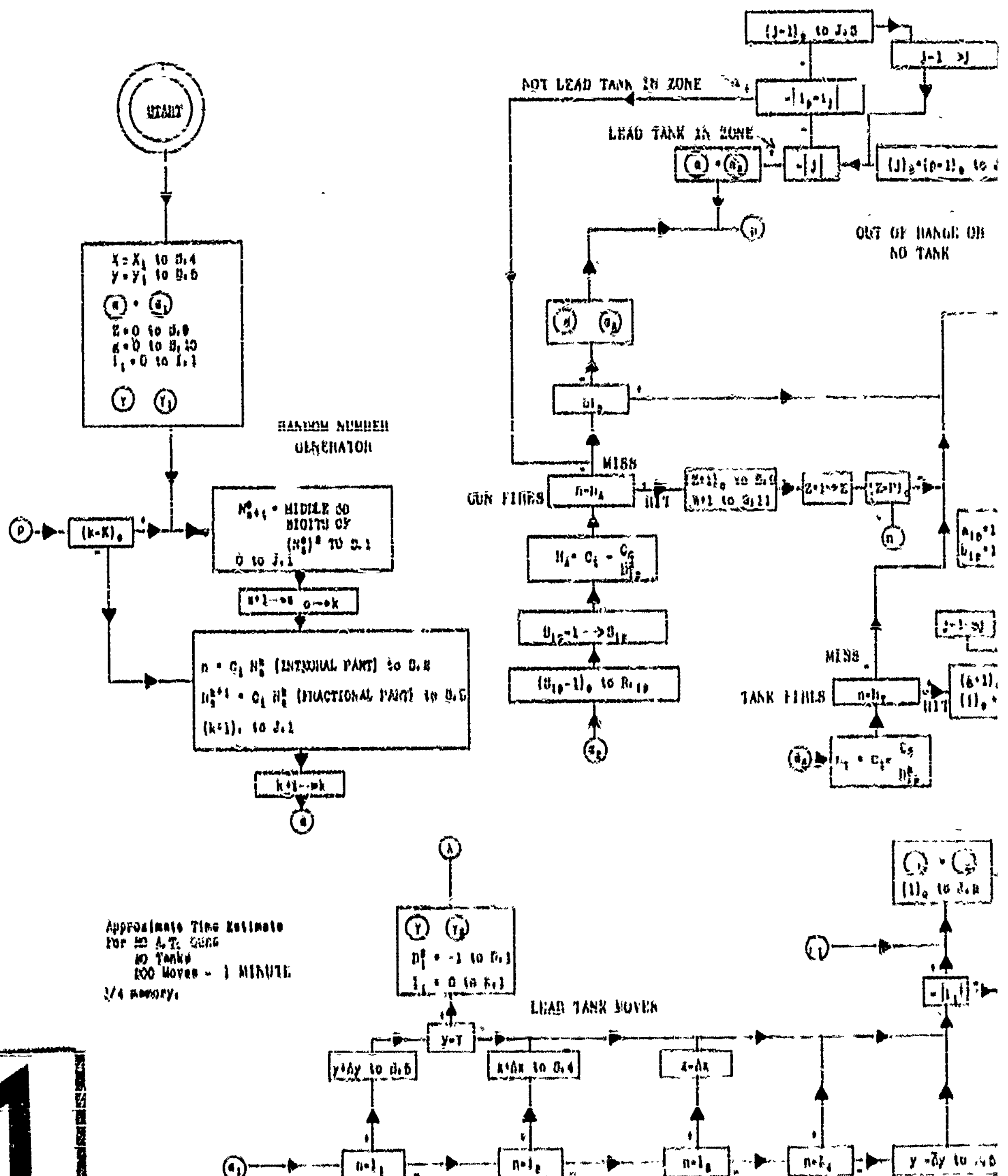
The range of the anti-tank guns and of the tank guns was taken to be 1000 yards. Inside this range the kill probability was usually taken to be a constant, 1/2, although variable kill probabilities were tried by hand.

The MANIAC flow diagram does not specify these probabilities but merely the method for using them. At the start of a game all probabilities associated with the movement of the tanks, the kill probabilities of the guns and the ranges can be given any values. The kill probability can be put in as an analytic function of the range if desired.

Key for Figure 3.

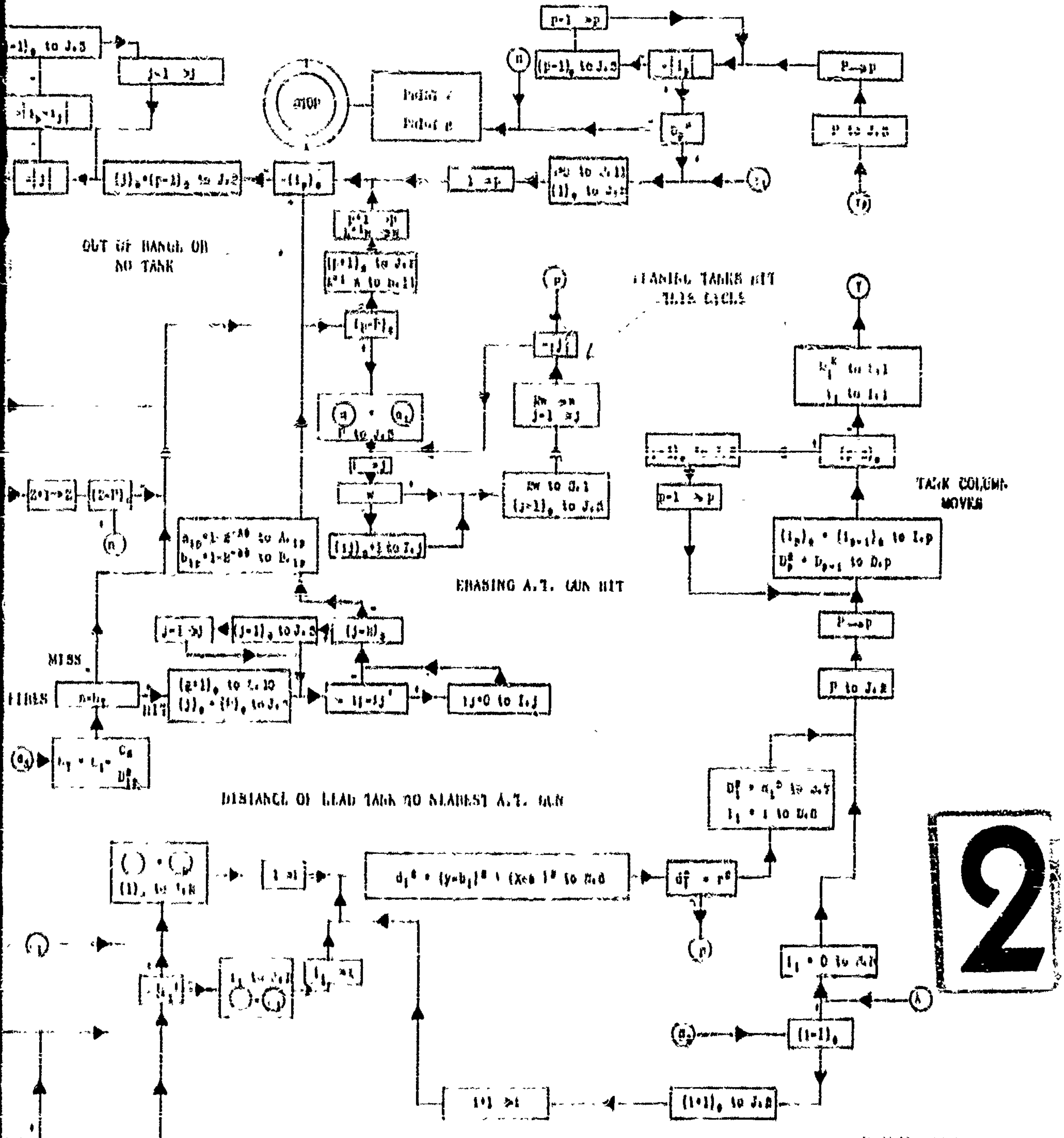
NOTATION

x	x coordinate of position of lead tank
y	y " " " "
x_0	Initial x " " " "
y_0	Initial y " " " "
N_s^k	kth fractional part of sth random number
n	Random integer used
C_1	Determines range of random integer
i	Index on A. T. guns
p	Index on tanks
I	No. of A. T. guns
P	No. of tanks
i_p	The number of the A. T. gun to which the pth tank is in range
d_i	Distance from ith A. T. gun to lead tank
D_p	Distance from pth tank to A. T. gun within range
r	Range of A. T. guns
a_i	x coordinate of ith A. T. gun
b_i	y " " " " "
Δy	Increment of forward motion
Δy	Increment of backward motion
Δx	Increment of motion to the right
Δx	Increment of motion to the left
l_1	Lower limit of n for forward move
l_2	" " " " " right "
l_3	" " " " " left "
l_4	" " " " " no "
H_A	" " " " " hit by A. T. gun
H_T	" " " " " hit by tank
Z	Number of tanks hit
K	Number of A. T. guns hit
S_{i_p}	" of shots by i_pth gun left before tanks get to shoot
Y	y coordinate of goal
w	A word used to remember which tanks have been hit, i.e., A "1" in the 10th digit position tells that the (P-9)th tank has been hit.


START
NOT LEAD TANK IN ZONE
LEAD TANK IN ZONE
OUT OF RANGE OR NO TANK
RANDOM NUMBER GENERATOR
MIDDLE 30 DIGITS OF
INTEGRAL PART
FRACTIONAL PART
GUN FIRES
MISS
HIT
TANK FIRES
LEAD TANK MOVES
Approximate Time Estimate
For 10 A.T. Guns
10 Tanks
100 Moves - 1 MINUTE
3/4 memory.

BREAKTHROUGH FLOW DIAGRAM

TESTING FOR BREAKTHROUGH

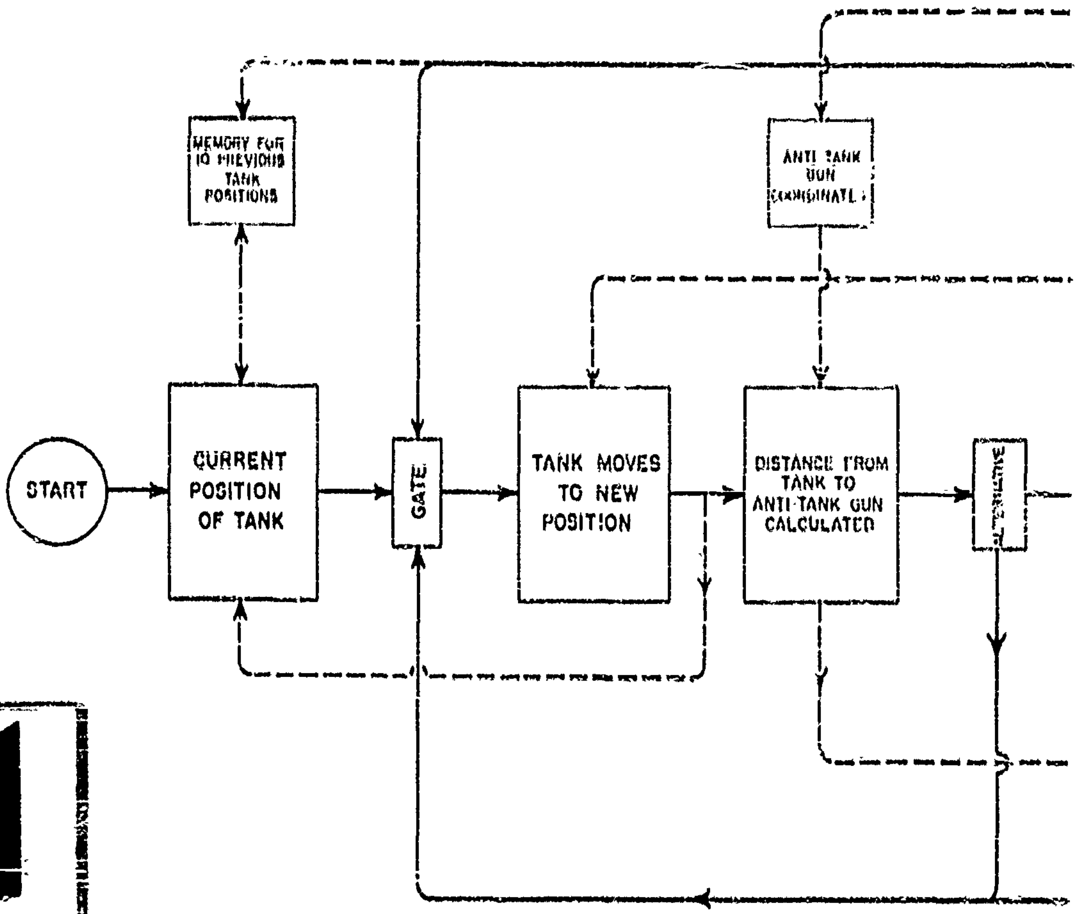


H. Wolle LASL
L. Herbst LASL

TIN SOL

TANK BREAKTHROU

Fig. h



CONTROL MOVES ON SOLID LINES

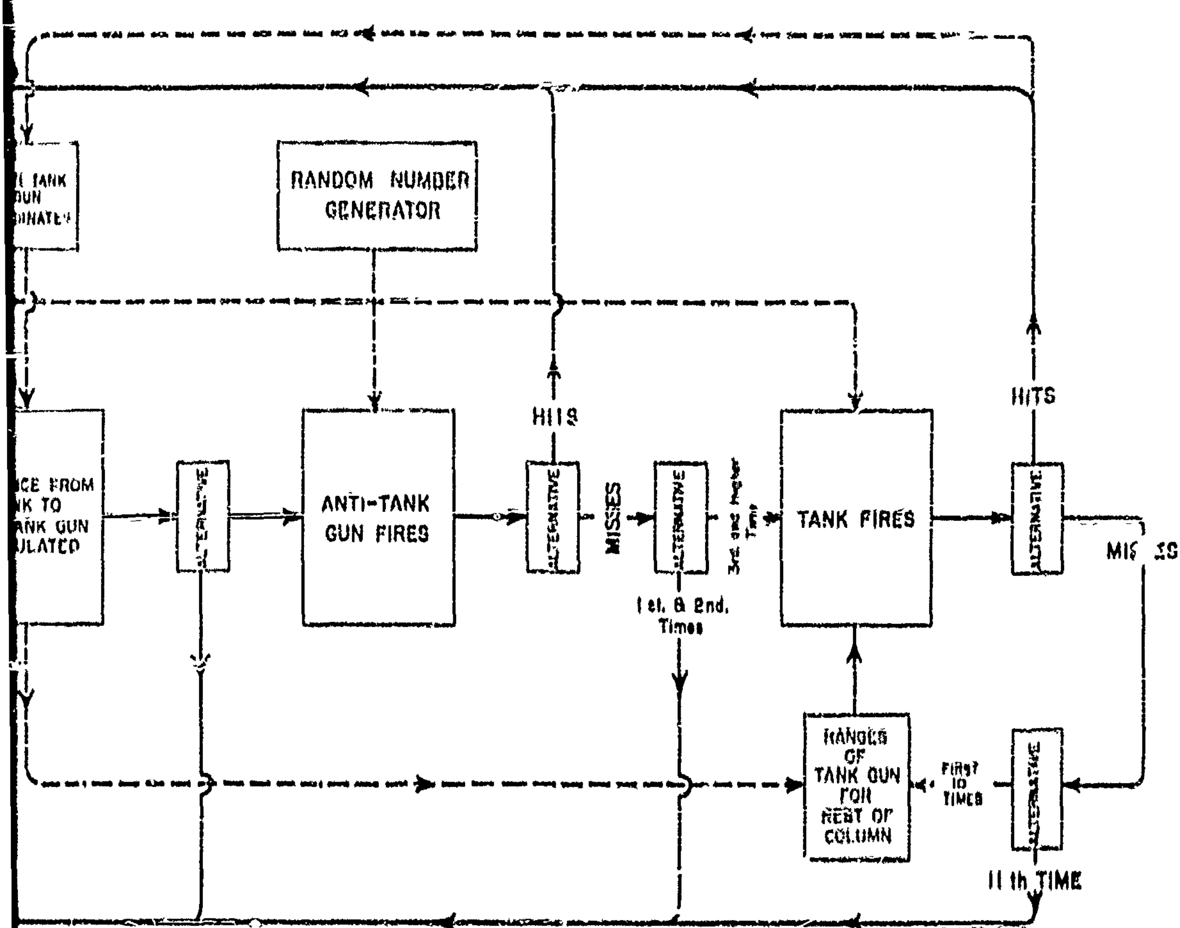
INFORMATION MOVES OVER DASHED LINES

10 tanks in a column
an area covered by A
fields of fire.

TIN SOLDIER

ANK BREAKTHROUGH PROBLEM

Fig. 4



2

O tanks in a column follow the leader and approach
n area covered by AT guns with no overlap in their
ields of fire.

PROGRAMMING A SET PIECE TYPE WAR GAME
FOR MACHINE CALCULATION

by

Richard E. Zimmerman

Reprinted from Unpublished Notes of the Author
Dated 2 April 1953

PURPOSE

A Set Piece type of war game will be programmed for machine calculation to investigate its applicability for evaluating tank effectiveness and as the first step in the program of Project ARMOR to extend the mathematical models currently in use to war games of considerably greater complexity.

FACTS

The set piece war game is intermediate in complexity between the simple battle models which are currently receiving much attention by Project ARMOR (and others) and the much more complicated models which is proposed would include the maximum detail that the fast modern electronic computers can process in times like 10 minutes per complete battle. The set piece battle is distinguished from other models by the fact that before the battle starts, the future position of the maneuvering elements is specified as a function of time up to the point that each maneuvering element must continue along its pre-set course unless it has been knocked out. This specification detracts from the realism of the battle, but it also makes the calculation of the battle very much easier.

The form of the battle here described follows generally that described by Col. Shanely, OCAFF, at a Theoretical Panel meeting at ORO, February, 1953.

The terrain used is imaginary in detail but is roughly similar to portions of Fort Knox.

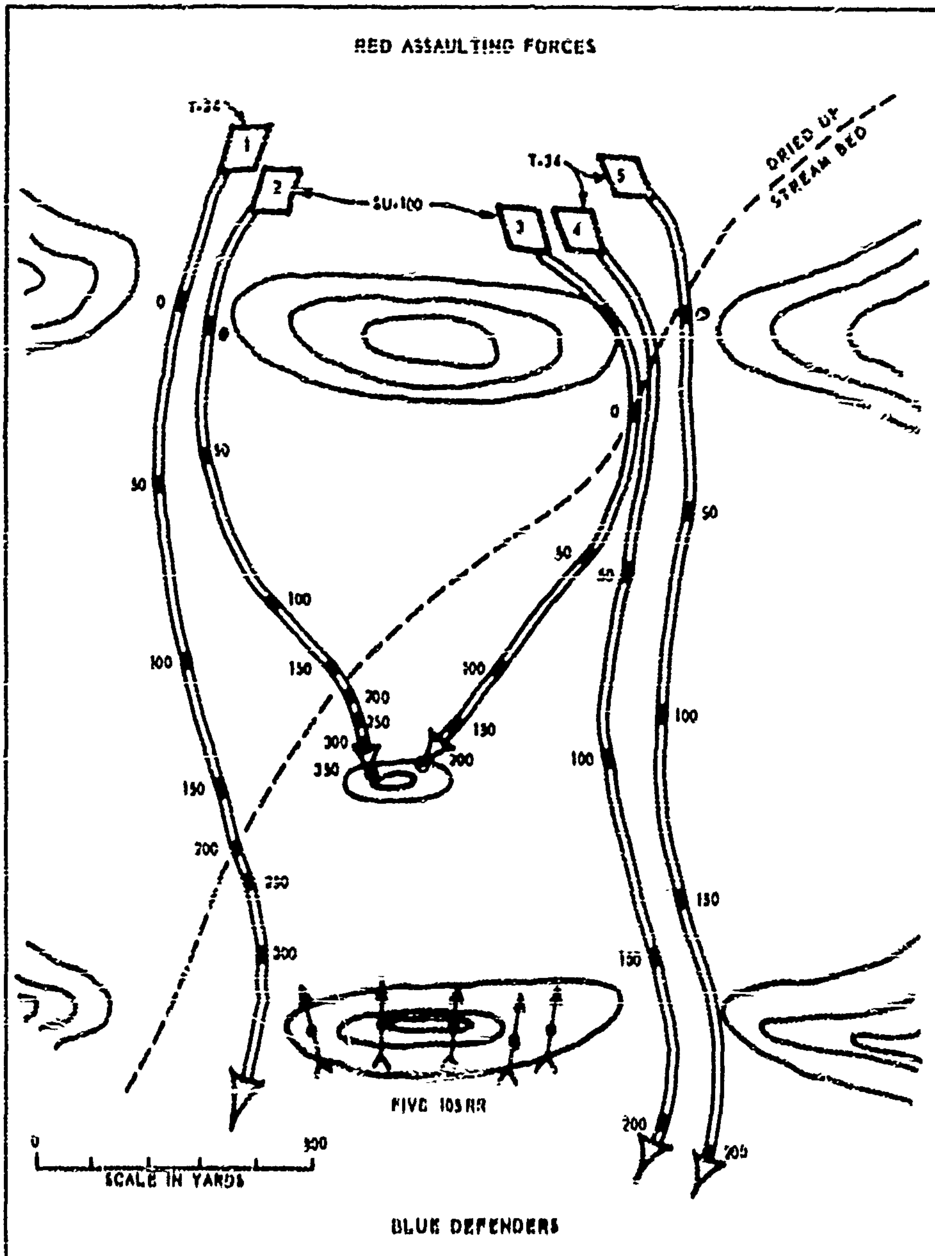
The type of battle considered will later be extended to other types of terrain and different missions involving tanks and anti-tank weapons, but retaining approximately the same degree of complexity.

THE BATTLE PLAN

Figure 1 shows the terrain and the disposition of forces involved. The Red forces at the top are presumed to make a frontal assault against the Blue forces entrenched at the bottom. While many weapons are expected to be firing on the battle field, we are considering in detail only the number of tanks killed by the anti-tank weapon and the number of anti-tank weapons lost to the combined weapon system of the Reds. The assaulting tanks themselves are not considered to bring effective fire to bear on the anti-tank 105 recoilless rifle until quite close, the order of 200-300 yds., since they are moving at high speeds. However, as soon as an anti-tank weapon is located it is assumed that a large number of weapons are brought to bear on it. This would include self-propelled guns overlooking the assault and mortars. Once the 105 RR has fired, the chances are high that the large flash would give his position away and his life would then be short.

THE DETAILED CALCULATIONS

To carry out the calculations for any given weapon pair--that is, one 105 RR vs one of the assaulting tanks--we consider in their proper order the following curves:



Battle Map for Set Piece Battle showing terrain, disposition of weapons and with the path of the maneuvering elements during the assault completely specified. The numbers along the arrows marking the path of the tanks give the time in seconds that the tank reaches that particular point in its advance.

FIGURE 1

(1) Curve giving the probability distribution of the first shot by 105 RR as a function of range to (nearest) tank.

(2) Curve giving survival probability of 105 RR which determines whether it has chance to get off its first shot.

(3) Curve giving kill probability of 105 RR against tank for this first shot.

(4) Curve giving probability of various time delays before 105 RR is able to get off each additional shot.

(5) Curve giving new position of tank after the delay in the firing of the 105 RR.

(6) Family of curves giving the survival probability of the 105 RR as a function of the time delay before the next shot, and of the range to the tank at the end of this delay.

(7) Curve giving the kill probability of the 105 RR against the tank for additional shots.

These curves must be completely specified before the battle starts for every weapon and weapon pair on the battle field which may become engaged. Some of the curves differ from weapon to weapon, while in other cases several weapons and weapon pairs may be described by the same curve in the interest of simplicity.

For machine calculation, the non-analytic curves must be transformed into a histogram. Since one will rarely get a satisfactory representation of the various probabilities in analytic form, the usual case will involve non-analytic curves.

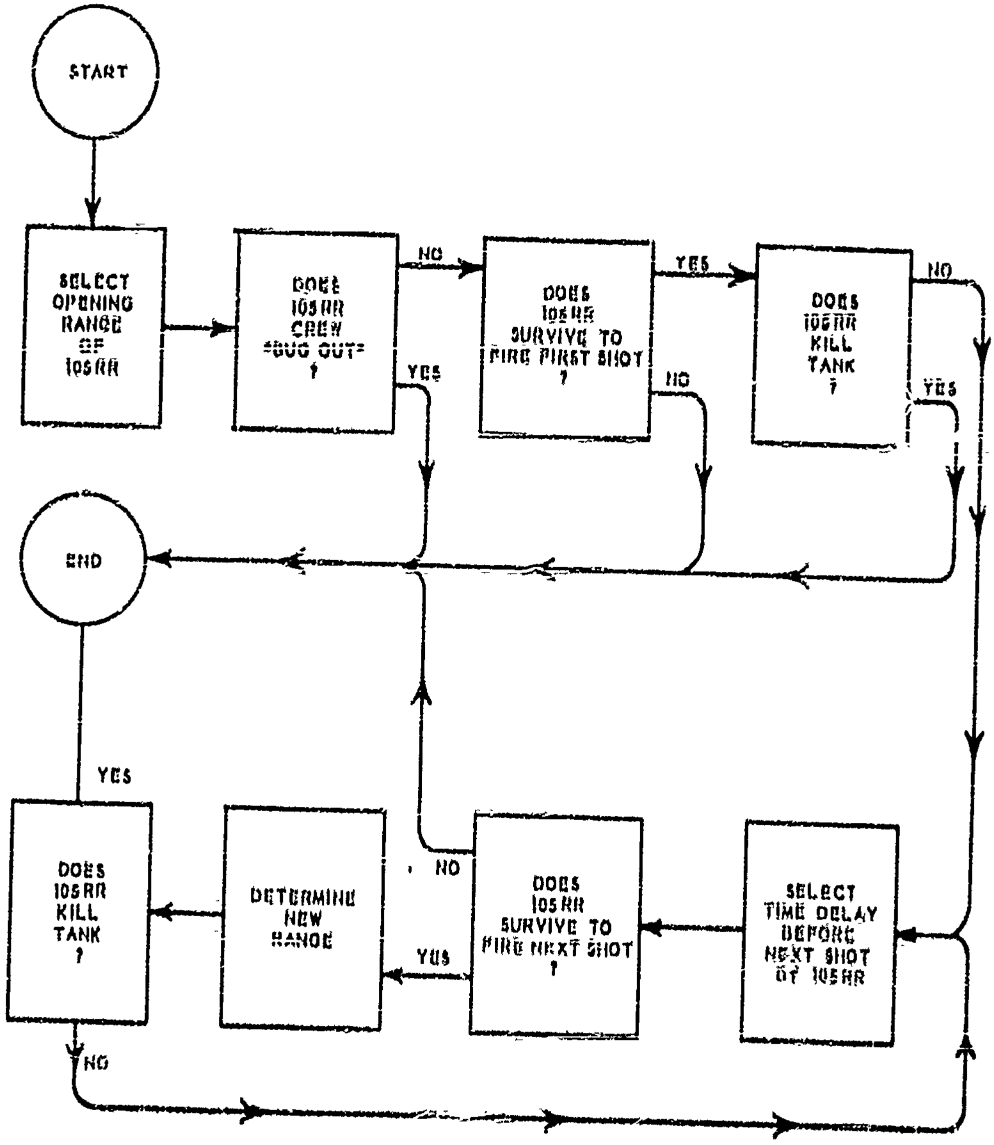
To describe the course of the battle most compactly, Figure 2 shows a logical flow diagram for the course of the calculations for one weapon pair. A complete battle will be composed of a number of such weapon systems considered one at a time.

Ideally each weapon pair should be considered for one very small increment of time before moving on to the next increment of time. Such an increment might be as small as one second. However, in the interest of calculational simplicity, the initial calculations will be made by considering each weapon pair up to the time of the first shot for each, then each weapon pair up to the second shot, etc.

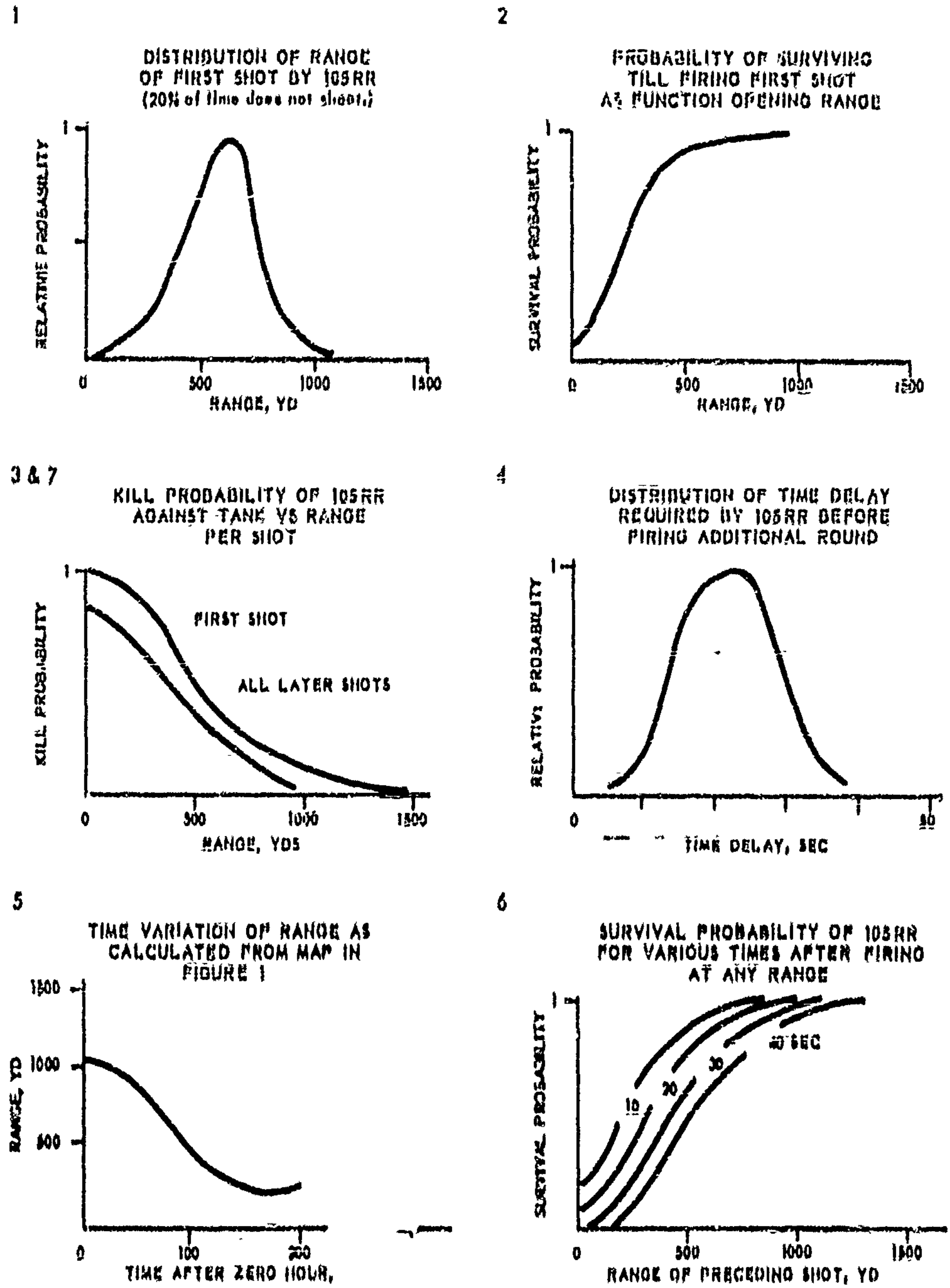
A HAND CALCULATION FOR ONE WEAPON PAIR

Figure 3 shows the general shape of the curves which were assumed for a hand calculation for one of the weapon pairs appearing on Figure 1, namely Tank No. 4, (a T-34) and the right-most 105 RR. Data were not available to define these curves in a really satisfactory fashion, but the methodology of this approach can be investigated using the curves shown. They were made as realistic as possible within the time limitations required for their production and the limits imposed by the methodology. Much more work will be required to develop fully satisfactory curves for these probabilities if the set piece type of battle is to be used to establish tank effectiveness.

The form of the hand calculations is shown in Appendix A. In it the curves have been broken down into a histogram with ap...



Logical Flow Diagram showing course of calculations for one weapon pair in Set Piece Battle.
FIGURE 2



Curves used to set up hand calculation for one weapon pair in Set Piece Battle

FIGURE 3

mately 10 "cells". The form shown for hand calculation is the same general form required for machine calculation using digital computers.

Some of the results of 74 hand calculations for this one weapon pair are shown below.

T-34 killed by first shot	44%
T-34 Killed by later shot	12%
Total	56%
105 RR killed before firing first shot	16%
105 RR killed after firing one or more shots	14%
105 RR crew "bugs out"	14%
Total	44%

The exchange ratio was 1.3 tanks knocked out per 105 RR committed. It will be noted that the curves used provided that the 105 RR crew should "bug out" 20% of the time, while the above results show a 14% "bug out" factor. The discrepancy is due to the small sample size (10 games gave this result) and it is not significant. Other results of the battles could be abstracted if they proved interesting, such as the distribution of the range at which the tanks were knocked out.

MODIFICATIONS FOR MACHINE CALCULATION

The details of the form described here are not necessarily the most efficient for machine calculation. The final form will depend upon the peculiarities of the particular machine used.

For the initial machine calculations, it is felt that one should consider the curves to be approximated by a histogram of

three cells, with 5 assaulting tanks and 5 defending 105 RR. This will reduce the initial coding problem.

APPENDIX A

RULES FOR HAND CALCULATION OF THE SET PIECE WAR GAME

We consider Tank No. 4 vs the right most 105 RR in Figure 1 as the AT wpn. Figure 2 gives the curves used in setting up these tables.

STEP I

Select opening range of AT wpn.

a. Choose a number at random between 0 and 99.

b. If a number is 0 or 1 AT wpn opens between 0 and 100 yds.

Number			AT wpn opens between		
0	or	1	0	and	100
2	to	4	100		200
5		8	200		300
9		13	300		400
14		20	400		500
21		29	500		600
30		41	600		700
42		56	700		800
57		64	800		900
65		70	900		1000
71		73	1000		1100
74		75	1100		1200
76		77	1200		1300
78			1300		1400
79			1400		1500
80		99	never fires at all, "bugs out"		

c. If wpn "bugs out", start over with next pair, otherwise proceed to Step 2.

STEP 2

Determine whether AT wpn survives long enough to get off first shot.

a. Choose number at random between 0 and 99.

b. If opening range (Step I) is

0	to	100 yds. AT wpn survives if no. is less than	15
100		200	36
200		300	53
300		400	67
400		500	79
500		600	88
600		700	93
700		800	97
800		900	99
900	or	over, certainly survives.	

c. If wpn does not survive, start over with next pair, otherwise proceed to Step 3.

STEP 3

Determine whether first shot from AT wpn killed the advancing tank by:

a. Choose a number at random between 0 and 99.

b. The tank is assumed to be killed if the number chosen is less

than	99	when the opening range is	0	to	100 yds.
	97		100		200
	93		200		300
	83		300		400
	67		400		500
	54		500		600
	43		600		700
	34		700		800
	28		800		900
	22		900		1000
	18		1000		1100
	14		1100		1200
	10		1200		1300
	6		1300		1400
	4		1400		1500

c. If tank is killed, start over with next pair; otherwise proceed to Step 4.

STEP 4

Determine time delay before AT wpn shoots again if it survives.

a. Choose a number at random between 0 and 99.

b. If number is 0 to 4, AT fires again in 10 seconds.

If number is	to	AT fires again in (seconds)
0	4	10
5	24	15
25	54	20
55	79	25
80	94	30
95	99	35

STEP 5

Determine whether AT wpn will survive to fire again.

a. Choose a number at random between 0 and 99.

b. Using the table below, pick the row corresponding to the range at which the last shot was fired and pick the number in this row which is also in the column for the time interval including the time delay before firing determined in Step 4.

c. If the random number is less than the number found in the table, then the AT wpn is assumed to have survived long enough to fire again.

Survival time in seconds after each shot at range R yds.

Range interval in yds of first shot	0-12	12-22	22-32	32-52
0-100	20	10	0	
100-200	40	20	10	0
200-300	50	30	20	10
300-400	60	40	30	20
400-500	70	50	40	30
500-600	80	60	50	40
600-700	95	70	60	50
700-800	99	80	70	60
800-900	99	90	80	70
900-1000		99	90	80
1000-1100			99	90
1100-1200				99
1200-1300				

d. If wpn does not survive, start over with a new wpn pair. Otherwise proceed to Step 6.

STEP 6

To find out how much the range has changed since the last shot:

a. Select from the range column below the range at which the last shot was fired.

b. Add the time delay selected in Step 4 to the time found in the time column directly opposite to the proper range.

c. The sum found in (b)--time in column plus time delay--will

then be opposite the range to the tank at the time of firing.

Time in secs.	Range in yds.	Time in secs.	Range in yds.
0	1170	105	425
5	1150	110	390
10	1125	115	350
15	1100	120	325
20	1080	125	290
25	1050	130	260
30	1015	135	235
35	975	140	215
40	935	145	205
45	885	150	195
50	840	155	190
55	800	160	186
60	760	165	180
65	725	170	180
70	690	175	175
75	655	180	175
80	620	185	160
85	575	190	185
90	535	195	200
95	495	200	215
100	460		

STEP 7

Finally, the kill probability is used to determine whether this last shot killed the advancing tank.

a. Select a number at random between 0 and 99.

b. If the number so selected is less than the number opposite the proper firing range (Step 6) than the tank is presumed to have been killed.

Range in yards		Probability no.
0 to	100	82
100	200	80
200	300	70
300	400	57
400	500	40
500	600	27
600	700	17
700	800	9
800	900	4
900	1000	1

c. If Tank is killed, start over with new wpn pair; otherwise continue the calculations by returning to Step 4 and repeat for the additional shots.

MAXIMUM COMPLEXITY COMPUTER BATTLE

by

Richard E. Zimmerman

Reprinted from Unpublished Notes of the Author
Dated 25 January 1955

ACKNOWLEDGMENT

The writer gratefully acknowledges the inspiration of G. Gamow (Consultant ORO) who first proposed the essential features of this methodology[1] and W. W. Nicholas who throughout pressed for the development of G. Gamow's proposal; to G. Kramer and E. Josephs of Engineering Research Associates who not only contributed their special technical and mathematical skills in applying the ERA 1101 computer to the calculation of these battles, but also aided in the development of the special computing techniques used; and to the many ORO staff members and consultants who contributed data, advice and encouragement. Special mention should be made of N. Smith (ORO) who led the early discussions which produced the guide lines applying to this study; S. Ulam (Cons. ORO) who contributed basic and original work on Monte Carlo Techniques; Col. Billingslea who supplied authoritative advice and proposals on the tactical aspects of the study; and V. MacRae and M. Grabau who kindly permitted the use of a quantity of original and unpublished tentative performance data for the armored vehicles.

MONTE CARLO COMPUTER WAR GAMING:
A FEASIBILITY STUDY
SUMMARY

MISSION

1. To develop a model of battle which will permit the simulation of a small unit combat action useful as a testing medium for new weapons, weapons systems, and tactical doctrine.

2. To apply existing techniques and originate new techniques rendering the model of battle subject to calculation by the largest electronic computers.

3. To make a trial calculation on a scale sufficient to test the technical feasibility of the model of battle and computing techniques.

FACTS

1. The rate at which unproved weapons of radical or unconventional nature are becoming available to our military forces is increasing tremendously, compounding the difficulty of evaluating new weapons and weapons systems in the absence of actual combat. Some of these weapons may strongly influence the organization and tactical doctrine of the military forces. Thus the effectiveness of all weapons, even those already tested in combat, may be altered. To be adequate, tests of weapons must be made in the context of the weapons system containing them. Proving ground data, while necessary, is not enough; yet full field tests of all proposed weapons is prohibitively expensive.

2. "Paper Analysis" of complete weapons systems has hitherto been inadequate to treat convincingly all essential elements of the system, particularly when the opposed weapons systems (1) are not directly comparable, weapon by weapon, (2) involve untested tactical innovations, or (3) are closely balanced. In the third case especially, the influence of fluctuations in the fortunes and misfortunes of combat on the outcome of the engagement has been largely ignored in paper analysis, there being no practical way to assess it using conventional mathematical techniques.

3. Recently a new technique for the solution of extremely complex problems, particularly those involving multiple probabilities, has come into being. This technique, called "MONTE CARLO," has been successfully used by mathematical physicists at Los Alamos and others to solve certain important problems which had been "unsolvable" by conventional techniques. The new technique (usually) requires that a large electronic computer be employed to carry out the calculations.

4. Large electronic computers are now available which in addition to their well-known ability to solve arithmetic problems at great speed, have also a capability for solving "logical" problems. That is, they can determine the logical consequences of a given set of facts and/or assumptions. They can be caused to make a decision to alter the nature of subsequent calculations in any manner desired as a result of a logical calculation.

DISCUSSION

From general considerations the scale of the combat operation to be treated in this feasibility study is shown to be properly of company size, and to involve tanks as the primary combat elements. It is argued that each tank must be considered individually in the calculations, hence that the details of the action must involve probability notions to fulfill the mission of the study. While conventional mathematical formulas appear inadequate to treat the complex actions of individual tanks, a recently developed computational technique -- "Monte Carlo" -- proves feasible and to have other advantages as well. From the requirement that Monte Carlo type probabilities be included in the treatment of the battle it is shown that the computer must complete the calculations for a single battle in between a few minutes to an hour, so that the series of battles required for a complete analysis (several thousand) will not take a prohibitively long time for computation.

A particular hypothetical combat operation is described which is consistent with the statement of the problem and which serves in the remainder of the report as the test vehicle for this feasibility study. The combat action selected was formulated with the aid of knowledgeable Army officers and civilian technicians. It is patterned after the "Reinforced Tank Company in the Attack" problem performed frequently at Ft. Knox to illustrate Armored Small Unit Tactics. The Ft. Knox problem has been modified in some respects, particularly in regard to the tank strength of the aggressor forces, which is here assumed to be approximately

on a par with those of the assaulting forces. The attacking forces include a medium tank company, 3 squads of infantry mounted in personnel carriers, with a battery of 4.2" mortars in support. The defenders are assumed to have a company of 10 medium tanks, a company of 5 SP guns and 9 squads of dismounted infantry.

The action is put in the context of an over-all tactical situation, and takes place on a piece of terrain patterned closely after an area a little over a mile square in Bavaria, 30 miles north of Würzburg. The major terrain features of this area are similar to the area at Knox where the attack problem is demonstrated.

The combat action is broken down into its essential elements of fire and maneuver. A precise statement of what calculations the computer must perform in order to accomplish the mission of this study is then formulated.

Stated briefly, these fundamental activities by the combat elements are (1) a decision to move from one small 100-meter x 100-meter square, which is its present position, to a selected neighboring square, affected by the factors of terrain and combat which must influence the selection; and (2) a decision to deliver a single unit of fire against a selected enemy target in accordance with the terrain factors and combat situation which must influence the selection and the effectiveness of the unit of fire.

These two fundamental activities are properly ordered in time by the computer, i.e., are caused to be performed in a sequence which makes military sense and at a rate consistent with the capabilities of the weapon and weapon crew.

The characteristics of computers in general and of the 1101 ERA computer in particular are described; and a technique is developed whereby the capabilities of the 1101 computer are applied to the solution of the problem of this study. The solution is seen to depend strongly on certain non-arithmetic, or "logical," calculations which the 1101 computer can perform with great speed.

A program is specified which causes the 1101 computor to compute in great detail the progress of the particular combat action under analysis, which involves directly the factors of terrain, communications, weapons, mobility and tactics that have been identified in the first part of the study as essential.

Every militarily significant portion of this program is described in detail and criticised concurrently to the extent feasible. The actual coding of the problem involves a careful arrangement of over 16,000 8-digit numbers. The rules used in translating the military statement of the problem into these "coding numbers," are described.

The results of 121 battles are analysed to determine the nature and accuracy of conclusions which might be derived from a series of battles were this program applied to a "real" situation.

The results of Monte Carlo computer battles appear in a form which raises certain statistical questions which must be investigated, and the present study provides useful answers to these questions. Essentially, the question is "How many times must a particular battle be repeated to give an acceptable measure of the 'average' outcome of the battle?" Related to the answer to this question is the spread in the results of a given battle; that is, the likelihood with which "unaverage" or exceptional outcomes of the battle occur. The study shows that 50 repetitions of the computer battle in its present form is sufficient to determine the "average" outcome with acceptable accuracy and shows that the spread of the battle results is fairly measured by the same number of repetitions.

A second important question is also investigated. For this purpose 50 additional battles were obtained for the case where the medium tanks of the assaulting force were replaced by a new set of tanks with (1) lower kill probability of its gun against the enemy armor, (2) an increased rate of (effective) fire, (3) higher vulnerability to the enemy armor, and (4) an increased mobility (speed of movement). The change in the numbers which specify these capabilities of the tanks follows roughly, but only in part, the difference between T-48 medium tanks and T-41 light tanks, and were derived from tentative performance data supplied by the staff of Project Armor. Of course, the results of the trial battles computed in this feasibility study cannot be taken as an accurate comparison of the effectiveness of the T-48 medium tanks with the T-41 tank. However, the comparison made is a clear example of an important application of computer battles. Such a comparison can be made as soon as a realistic battle code is devised and accurate performance data is available.

An additional 21 battles were computed for the case where "heavy" tanks replaced the mediums. The performance data for these tanks followed roughly tentative performance data for the T-43 tank supplied by Project Armor staff.

In both cases, the modified tanks caused a variation in the outcome of the "average" battle which was measurable with useful accuracy and gave rise to a spread of results which was not so wide as to make predictions impossible, nor so narrow as to cast doubt on the methodology.

A description of an improved computer battle making use of the techniques developed in this study is given, which appears to be feasible on new computers now available, and which is formulated with sufficient detail and realism to permit its application to the solution of real and pressing problems relating to the T/O&E of small combat units.

It builds directly on the lessons learned from this feasibility study, and takes into account all the major improvements in the military realism which appear necessary for the useable computer battle code. In particular much greater flexibility in the tactical doctrine governing the actions of the combat units is provided for. The command-control structure of subordinate units is an integral part of the proposed code and permits inclusion of the important command control problems in a realistic fashion; including change of combat mission and of the tactical means adopted for the execution of the mission during the battle. Since such command decisions are made on the basis of the commander's knowledge at the time, the operation and effectiveness of the commander's data-gathering system, including his radio net, are a part of the proposed computations. With this addition, the methodology appears capable of being directly extended to involve combat units of battalion size and larger, as soon as detailed performance data is available.

A further application of the methodology is proposed which uses it as an adjunct to CPX type map exercises where the computer replaces to a substantial degree the umpire system currently in use. Such a system would appear to have major advantages, since it would permit much more accurate and detailed assessments of changing capabilities and casualty rates without distorting the time scale of a realistic CPX. While this application can be made using existing equipment, full exploitation of this technique requires an improved system for the controling and directing of the computer calculations and for the presentation of the results of the computations to the participants in the CPX. Recent developments in "television type" data display equipments show great promise for such application.

CONCLUSIONS

1. The Monte Carlo computer war gaming methodology is technically feasible for small unit actions within the restrictions imposed by time and cost factors.

2. All significant factors affecting combat actions which have been identified, can be included in computer battles of this type.

3. The outcome of this type of computer battle is sufficiently sensitive to the capabilities of the combat elements taking part in the battle to yield significantly altered battle outcomes when realistic variations in the combat capabilities of the weapons are made.

4. The methodology can be extended to include larger combat units, additional factors when they become known, an increased variety of units, or any factor which can be precisely described.

5. The methodology is ideally suited to permit direct participation in — and concurrent criticism of — the studies by nonmathematical personnel — most importantly, by military officers with extensive combat experience.

6. The methodology may be employed in almost its present form with existing equipment to vastly improve OPX type exercises, either for their training or research value.

7. The OPX type operation may be made much more convenient if existing equipment be modified for use as visual display equipment operated directly by the computer.

MONTE CARLO COMPUTER WAR GAMING: A FEASIBILITY STUDY

INTRODUCTION

This memorandum is a report on the feasibility of a new method for more completely analyzing the effectiveness of weapons, weapons systems, and tactical doctrine. It should greatly increase the number of military problems which are susceptible to scientific analysis. The method extends such analysis to include important factors of terrain, mobility, and command-control problems in a detail not hitherto practicable.

MISSION

The new method essentially involves causing a large electronic computer to "simulate battle." The goal of the over-all program -- for which this report is a feasibility study -- is to produce a set of general instructions for an electronic computer which will enable the computer to calculate the progress and outcome of a combat action involving, within wide limits, (1) any desired weapons, weapon systems or other equipment, (2) any specified level of proficiency of officers and men, (3) any specified tactical doctrine and mission, and (4) any selected conditions of terrain, weather, and over-all situation. It is highly desirable (if not absolutely necessary) that these factors be described in such a way that variation of any of these factors of men, weapons, and terrain be merely a matter of changing certain characteristic numbers at the start of computation.

So that the feasibility study may be considered in concrete terms at every stage of the discussion, a particular combat action is fashioned at the outset for trial analysis.

The feasibility of the method does not depend upon the details of the example selected.

The usefulness of a "battle simulator" in analyzing the effectiveness of new weapons, weapons systems and tactical doctrines, is self-evident. With increasing numbers of new weapons with radical or improved capabilities becoming available to all military forces, the problem of assessing their true worth becomes ever more difficult. Tactical and organizational innovations which may appear desirable to fully exploit new weapons may cause unexpected chain reactions throughout the organization which could nullify the expected improvements. As the tempo of battle steps up, the command-control-communication system becomes ever more critical. As the weapons themselves become more complex the nature and degree of logistical support and training required acquire a critical bearing on the selection of the best weapon system.

Faced with a potential enemy with essentially different resources at his command, the comparison of US military capabilities with Soviet capabilities appears in its most intractable form, since dissimilar forces must be compared.

Finally the potential violence of the initial stages of combat puts the most severe requirement on the thoroughness and accuracy of weapon-tactical analysis in advance of that combat.

More than ever before, time may be lacking after combat begins to correct errors revealed by the test of combat.

The problems requiring analysis which are discussed above are in the form least susceptible to conventional analysis. Summarizing, their general characteristics are:

(1) Tactical and organizational innovations involved.

(2 Increasingly critical interdependence of weapons — communications — logistics and training.

3. Comparison required between dissimilar military forces and among dissimilar tactical and strategic alternatives.

4. Stakes are higher

The present study has as its primary mission to provide an improved methodology for dealing with problems of these types.

In the following sections, BASIC FACTS, SCOPE, AND ASSUMPTIONS, this memorandum derives the general restrictions which limit, or influence, the type of combat action and the way it is analyzed.

There follows in the section, MILITARY DESCRIPTION OF BATTLE, a description of the trial combat action to be analyzed and the reasoning used in reducing the combat action to its basic components of fire and maneuver.

The next section, THE COMPUTER BATTLE, applies this reasoning and the capabilities of the computer and completely describes the way in which the computer simulates the trial combat action.

Following this the section, RESULTS OF TEST BATTLES, gives the results of 121 trial battles carried out by the computer. The lessons to be drawn from these results are described.

In the next section, PROPOSED APPLICATION TO T/O&E and TACTICAL STUDIES, a modified and much improved model of battle is described which appears capable of simulating the battle action of small units to a degree sufficient to warrant its use in future analysis.

The last section, CONCLUSIONS, states the conclusions to be drawn from this feasibility study. Because the present memorandum is exclusively concerned with describing a new methodology, that is, a new way of carrying out military analysis, there can be no formal recommendations for action, as there are in memoranda which present a solution to some particular problem.

FACTS, SCOPE, AND ASSUMPTIONS

The discussion which follows has the purpose of identifying the necessary and desirable features of the proposed methodology in general, and this feasibility study in particular.

In the first section -- Scope -- the necessary scale of the combat action to be used in developing and testing the methodology in this feasibility study is derived. In the second section -- The Basic Assumption -- is developed the point of view applied throughout the study. In the third section is derived the length of time the computer may be permitted to calculate a given battle.

Finally, the general characteristics and capabilities of selected electronic computers are described.

With the end of this section the general characteristics of the methodology and the manner of its testing and use are established.

Scope

This feasibility study should use the simplest possible trial combat action. On the other hand, the combat action to be analyzed must be large enough to be self-contained; that is, it should include all the factors which influence the battle once the forces are joined. This means that, if the action is to include tanks, the battlefield must be large enough to include all, or most, of the elements which interact with the tanks. That tanks should be included in this study is suggested by these considerations:

1. Tanks represent the largest capital outlay and one of the major logistical problems of the Army.

2. The cost of carrying out analyses of the type proposed by this study is probably justified only for the most pressing and important problems.

3. Tanks are the combat elements most severely restricted by their mechanical characteristics and thus are more susceptible to mathematical analysis.

If tanks are to be included, then the smallest self-contained battlefield will be of a size comparable to the maximum effective range of their guns, i.e., about 1 to 2 miles on a side.

A reasonable combat action on such a battlefield could involve about a company of tanks but hardly a smaller unit. Since the smallest reasonable action is desired the above discussion fixes the scale of the action.

Even at this small scale, operation of tanks without infantry is improper, and some indirect fire weapons should be included.

A complete combat action on such a battlefield could conceivably be completed in as little as 30 minutes, if the action were of sufficient intensity. In this case the action would not involve logistical problems during the action and they could be properly left out of this first study.

Similarly, air strikes could be left out of this first study, since their influence is more easily assessed by over-all considerations. Also the much larger range of TAC aircraft means that assessing the influence of alternate allocations of strikes during an assault, requires that the combat action be of much larger size.

These considerations therefore define the combat action which should be analyzed as the test vehicle of this feasibility study.

Summarizing, the combat action should

1. emphasize tanks;

2. feature intense action -- lasting about 1/2 hour;

3. include company-sized units with reasonable attachments of infantry and indirect fire weapons;

4. take place on battlefield of one- or a few-square miles.

The Basic Assumption -- MONTE CARLO

The most basic assumption made in developing this methodology is that, to be a successful battle simulator, the model of battle used must refer directly to the individual participants in a combat action -- at least so far as the major combat elements are concerned. In the previous section, tanks were selected as the combat element to be emphasized in this feasibility study. Thus, it is assumed to be necessary to treat the tanks individually. That is, their movement, firing, and all other actions must be treated as individual and separate actions -- not averaged out over a platoon or other tactical unit.

This assumption appears attractive for at least three reasons. Firstly, the physical characteristics of weapons are (usually) best determined on an individual basis and are (usually) the most accurate information available. The results of calculations starting from such data are apt to be more believable than calculations starting from less well-known data.

Secondly, the proposed methodology will be the more flexible, the more readily are weapons and equipment added, altered, or removed from the weapon systems. It is more convenient to do this when the battle model includes the weapons and their characteristics explicitly, than when weapons and equipment must be combined in some average way before insertion into the model of battle.

Finally, one of the primary purposes of constructing this new model is to render the interactions between weapons susceptible to calculation. Thus to the extent that these interactions are "averaged out" prior to insertion into the model, they are not subject to analysis and part of the purpose of the methodology would be frustrated.

Some compromise must be expected in this regard. The computer does not have an infinite capacity to treat all weapons and other equipment separately. The compromises which will be necessary in this connection will grow out of specific limitations of the computing machinery. They will be considered only as and when the need arises.

Summarizing, the basic assumption states: "as many as possible of the weapons and equipment entering the battle should appear separately and distinctly in the model."

When describing separate actions of an individual combat unit, e.g., a tank, it appears inescapable that probabilistic notions are required. Thus, with a given round, a tank will either hit an enemy tank or it will fail to do so. The difference between various tanks in this regard can only be in the probability of a hit.

It will now be argued that it is a natural consequence of the basic assumption stated above, that the use of conventional mathematical formulae is not feasible for constructing the desired model of battle. Or more properly, there is a feasible alternative to the use of conventional mathematical formulae. Reduced to its simplest form, the argument is that the use of mathematical formulae is a shorthand for describing an observed or proposed regularity in nature; but this is precisely the factor which is currently lacking in considering the influence of new weapons, new weapon systems, new tactical doctrines and the dissimilar military resources of the opposing powers. It is to discover new regularities in large military operations that a new methodology is required.

There are many cases where the regularity of the laws of combat are clear enough to warrant the use of mathematical formulae; it may be that future research and analysis will uncover even more extensive regularities in combat of a type which can be accurately described by convenient mathematical shorthand. But the express purpose of the new methodology is to extend analysis into complex areas which have not yet been tractable to conventional analysis.

This point of view would not be a very useful one if it were not that fast electronic computing equipment now makes feasible a nonmathematical technique for analysis which does not require the same degree of observed regularity in combat. This technique is called "MONTE CARLO." It has been used successfully for the solution of certain very complex systems of interest to nuclear physicists. These systems are characterized by very complicated probability "equations," comparable to the probabilistic concepts which will be found necessary to describe the actions of individual tanks on a battlefield.

The essence of Monte Carlo type calculations is easily understood. A simple example is possible in connection with umpire decisions for determining the progress of a battle in a CPX map exercise. Thus if the outcome of some small action within the CPX is considered by the Umpires to be known in terms of a probability but not worth further detailed study, they might make their decision on the basis of flipping a coin. For example, in a CPX at Corps level, many individual companies may be given a 50-50 chance of taking their objective on the first day. A decision on whether each company did reach its objective might then be made by flipping a coin for each case. If there were a large number of such 50-50 propositions, the feeling might be that the influence of chance in the coin flipping would

average out in the long run. It is more unlikely that umpires would be willing to use such a means for decision if there were only a few such cases.

This is an example of Monte Carlo type calculations. A computer can be caused to do something very similar to flipping a coin. It can also "use loaded coins" associated with any desired probabilities or odds. Appendix A reviews the most important background of Monte Carlo type calculations and goes into some detail about the method used for causing an electronic computer to make such calculations.

There is a second reason for developing a methodology which uses Monte Carlo calculations. The command-control-communication-decision process necessarily included in a battle simulator -- a "Monte Carlo Computer Battle" -- is a system which intimately involves human thought processes. Human reasoning appears to depend more on a system of "logical computation" than on an arithmetic or mathematical system. It will be shown that the Monte Carlo system of calculation is readily applied to a system including many logical (human) thought processes; while a methodology depending basically on conventional mathematical formulae appears very inadequate.

Again a compromise between the use of Monte Carlo operations and conventional mathematics will be found necessary from time to time. Such compromises will result from the limited capacity of the computing machinery. They will be discussed and resolved in each particular case, as and when the need arises.

Time Limitations

The one remaining major restriction on the scale of the trial combat action is the length of time which can be allowed for the computer to fight through a single battle. This does not depend upon details of the methodology. It depends only upon the way in which the methodology may be applied to the solution of military problems and upon the decision to use Monte Carlo type calculations.

Appendix A gives a detailed description of the manner in which the proposed methodology would be applied to a thorough analysis of the influence of some given factor in the battle. In the example discussed in Appendix A it is supposed that the factor under analysis is the type of gun to be mounted on a tank, when all other major design features of the tank have been fixed. The only factors to be varied in the series of computer battles are those directly relating to the choice of gun. These might be expected to be (1) killing power of gun, per shot, as function of range and target type; (2) rate of effective fire; (3) size of base load of ammunition.

One procedure for this application would be to determine the variation in the "effectiveness" of the tank in one or more combat situations as the "power" of the tank gun is varied. Supposing that an acceptable measure of "effectiveness" if available -- this might be the average number of enemy tanks killed by each friendly tank* -- and supposing that the variation in

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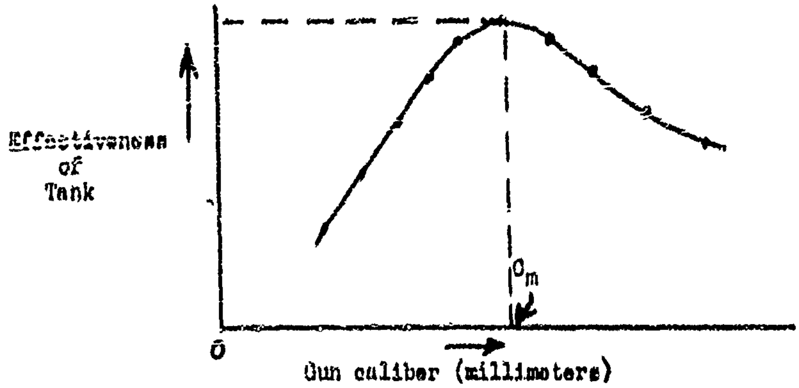
* This is the definition of the "effectiveness ratio" used in ORO-T-313 by M. Grabau and V. McRae.

gun power is properly measured by gun caliber alone, the results of a series of computer battles might be expressed by the following type of graph.



Such a result would indicate the "most effective" gun for the tank, in the particular combat situation used, should have a caliber of "C" millimeters.

If other combat situations lead to different "best" choices of gun caliber, then a compromise would be required.

It is assumed that the complete analysis can consume no more than 6 months to a year. Certain additional assumptions are made about the number of battles required for each trial of a given tank gun, including variation in the tactical situation as well as additional calculations which tend to confirm and check the results.

The over-all program required to determine the military consequences of a variation in tank gun type is therefore shown to involve perhaps 10,000 repetitions of the battle. It follows that each battle can consume no more than a few minutes for its calculation, on the computing machinery used for the analysis.

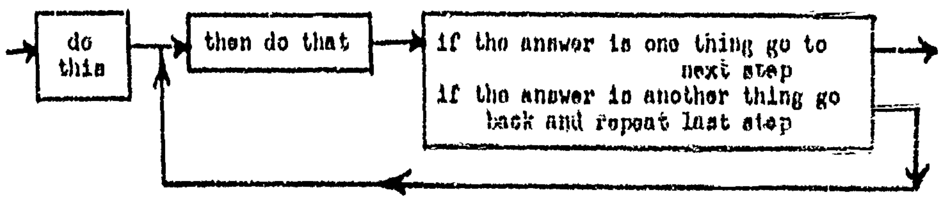
It should be emphasized that the example discussed in Appendix B is not the only way in which the methodology may be applied to analysis. Instead it represents what is thought to be the most extreme case among possible applications and therefore results in the most stringent limitations on the time which should be available to the computer.

Since the computing machinery which will be used in the application of this methodology is at least 10 times as fast as the ERA 1101 computer used in this feasibility study, a time limit 10 times larger than that calculated in Appendix B can be used. Thus the average computer battle on the 1101 computer should be completed within 20 minutes. Since the basic assumption was to insert as much of the battle participants and equipment as was possible, no lesser time will be considered for purposes of this feasibility study.

Capabilities of Electronic Computers

Certain military facts and analytical assumptions have been described which strongly influence the methodology. The nature of the proposed methodology also depends critically on the capabilities of the computer.

Nature of Computers in General. The essential difference between a desk calculation machine and the electronic computers used for computer battles is the "automatic" nature of the latter. That is, an electronic computer not only can add, subtract, multiply and divide, but it can also be instructed to perform a long series of such operations in any desired sequence with no further attention required from the human operators. It can be instructed to carry out such an extensive number of mathematical operations that a special means -- a "flow diagram" -- must often be used to show clearly the order in which these mathematical operations are performed by the computer. A flow diagram will have the following general character:



Each block indicates some simple computation which the computer must carry out. The arrow(s) leading from a box then indicate the next operation. By following along the arrows in a flow diagram every step of the computations can be traced out. In principle, such a flow diagram relates only to the logical structure of the computation itself, and not at all to the particular computer for which it is designed. In practice, however, the particular way in which the over-all problem is broken down into simpler parts, will depend upon the special characteristics of a particular computer.

The computer itself can be described generally without detailed reference to the "nuts and bolts" of its construction. Thus all general-purpose electronic computers can be considered as composed of 4 functional subgroups. These are:

1. The ARITHMETIC unit(s). (Where the adding, subtracting, etc., is actually carried out)

2. The MEMORY unit(s). (Where numbers are retained before, during, and after use)

3. The CONTROL unit(s). (The source of the instructions which tell the ARITHMETIC unit(s) what to do next, where to get the numbers which are to be used, and where to store the answers)

4. The INPUT-OUTPUT units. (The machinery used by the human operator to tell the computer what to do and which numbers to use; and the machinery used by the computer to "tell" the human operator what it has done, and what the answers are)

These functional units are usually, but not necessarily, associated with separate electrical or mechanical units. In the case of the ERA 1101 computer used for this feasibility study, the physical equipment which perform the above functions are:

1. ARITHMETIC UNIT: About 600 ordinary (radio) vacuum tubes.

2. MEMORY UNIT: A rotating cylinder, covered with a magnetic substance similar to that used on magnetic tape (radio) recorders. Records voltage pulses, interpreted as numbers. Capacity, 16,384 7-digit numbers.

3. CONTROL UNIT: About 400 more ordinary (radio) vacuum tubes.

4. INPUT-OUTPUT UNIT: INPUT is by paper tape having holes punched in it by a special typewriter. OUTPUT is by the same type of paper tape and/or a direct connection from the computer to a fast electric typewriter.

The possibility and advantages of using other computers with additional equipment carrying out these 4 functions will be considered in the last section — PROPOSED APPLICATION OF METHODOLOGY TO T/O&E AND TACTICAL STUDIES.

The computer can be caused to perform any stated sequence of arithmetic operations (add, subtract, multiply and divide) and certain special forms of these arithmetic operations, usually called "logical" operations. These will be more completely described in later sections.

The exact procedure followed in the use of the computer consists of the following stages:

1. The military engagement is broken down into simple understandable steps.

2. Each step is translated into an equivalent mathematical or logical operation which the computer can perform.

3. A number code is prepared which will cause the computer to carry out all the calculations in the desired order and which includes all the numbers necessary for the calculations.

4. A paper tape of some length is then punched by typing the number code on a special typewriter.

5. The prepared paper tape is fed into the computer which then starts its calculations.

6. Selected results of the calculations are caused to be typed out directly onto a special typewriter as they are obtained. At the same time a more detailed record of the calculations is also punched out by the computer on paper tape which can be inspected at a later time.

However, this more detailed account of the calculations cannot be conveniently "read" directly. Usually the paper tape must be re-run through the computer while the computer re-interprets what it had originally punched out on the tape. In so doing, the computer can directly cause the special typewriter to type out the detailed results which were stored in the tape.

In terms of this memorandum, Step 1 will not have been completed until the complete "scenario" of the battle has been described.

Selected performance data of the computer is given in Table I.

TABLE I

PERFORMANCE DATA ERA 1101 COMPUTER

Memory Capacity - - - - - - - - - - - - - - - - - -	16,384 7-digit numbers
Max. Additions (sub.) per second - - - - - - - - -	10,000
Max. Multiplications per second - - - - - - - - -	3,000
Total number possible distinct operations - - -	50
Time to fill memory from tape - - - - - - - - - -	7 minutes
Digits typed out per second (or alphabetic char.)	7

Computer Calculation. The significant types of calculations, or operations, which the 1101 computer can perform may be listed under three general categories.

1. ARITHMETIC OPERATIONS: Ordinary addition, subtraction, multiplication and division.

2. LOGICAL OPERATIONS: A special form of arithmetic, designed for carrying out a type of calculation akin to "logical reasoning."

3. "JUMP" OPERATIONS: A special class of operations which makes it possible for the computer to alter its type of calculation depending upon the numerical result of some previous calculation.

Every automatic calculating machine has at least a few operations of each type listed above. In addition, there are other less interesting operations, which stop and start calculation of the computer, cause the computer to punch or type out selected results, to "read" numbers punched into the INPUT tape, and perform other necessary but subordinate functions.

Since the ERA 1101 computer has a definite list of possible operations, every step in the computer battle is ultimately stated in terms of one or a few of these operations.

The computer battle described in this memo makes important use of all three classes of operations. In general, any calculation which is expressed in terms of a logical operation could also be reduced to an arithmetic one. However, tremendous savings in time and memory capacity, as well as an increased simplicity of conception, is possible when logical operations are used. Much attention is paid to those matters in subsequent sections of this report.

For the present, a simple example of each of these three basic types of operations is given, after which the combat action is described, dissected into its components and finally translated into a series of instructions for the computer in terms of these basic operations.

<u>A simple arithmetic operation</u> which the computer could be caused to carry out might be: "Add the number of Blue tanks killed (number is stored at place X in memory) to the number of Red tanks killed (number is stored at

place Y in memory) and store the sum at place Z in memory. It takes 3 separate steps for the computer to perform this simple addition.

Step 1. Take number stored in place X and put into "adder."

Step 2. Take number stored in Y and put into "adder."

Step 3. Take sum now in adder and put into place Z.

There is a special number code which is used to cause the computer to perform each step.

A simple Logical Operation which the computer could be caused to carry out might be "there is a number, composed of 5 digits which may be either 1's or 0's, e.g., 10110. The first digit is a '1' if the first tank has been killed, an '0' otherwise." The second digit is a "1" if the second tank has been killed, it is a zero otherwise; and so on for the interpretation of each position in the number; with the third digit relating to the third tank, the fourth digit to the fourth tank and the fifth digit to the fifth tank. This 5-digit number is stored at the place X in the computer's memory. Question: is the third tank dead?

Step 1. Take number in X and put in "adder."

Step 2. Take number in Y (number is 00100) and put into adder; form the "sum" of the number in X with the number in Y using the convention that the number expressing the sum will contain a 1 in a given position if both the number in X and the number in Y have a 1 in that same position. Otherwise the digit in that position in the "sum" is to be a zero. Carrying this out shows:

number in X	10110
number in Y	00100
"sum"	00100

Step 3. Is the "sum" in adder different from zero? If it is then the third tank is dead.

The above example identifies the number resulting from combining the number in X with the number in Y as a "sum." Logicians call it the "logical product." This type of operation plays an extremely important role in the computer battle. It is discussed in much greater detail in the next section. There is a special number code which causes the computer to [illegible] three steps in about $\frac{1}{10,000}$ of a second.

<u>A simple "jump" operation</u> which the computer may be caused to calculate might be "stop the computations and type out the letter R if the total number of tanks killed (calculated in the above example of an arithmetic operation) is as much as 17." The steps for carrying out this calculation are listed below.

Step 1. Put the number in place U (this is the number 17) into the adder.

Step 2. Subtract the number in place Z (this is the total number of tanks killed computed in the previous example of an arithmetic operation) from the number in the adder. The adder now contains the difference between the number in U and the number in Z.

Step 3. Test the size of the number in the adder If it is exactly zero go to Step 4. If it is not zero go to Step 5.

Step 4. Cause the typewriter to type out the letter signified by the number in place W. Then stop the computer. (Note. The place W must have inserted into it before the start of the computations that number which will cause the typewriter to print "R".)

Step 5. Go to the next proper step for continuing the battle.

There is a precise number code which will cause the computer to carry out these steps in as little as $\frac{1}{3000}$ of a second.

A further discussion of the various operations of the computer will be deferred until after the military action itself is described. At that time it will be necessary to consider the capabilities of the computer in more detail, since important alternatives in the way the battle is treated by the computer depend critically on these capabilities.

SUMMARY

This completes the consideration of the general features and restrictions of the proposed methodology. The remainder of the study will develop the methodology within the limits imposed by the facts, restrictions, and assumptions now identified.

COMBAT ACTION AND ITS ANALYSIS

A combat action is described which fits the general requirements developed in the introduction. The action is analyzed into its most elementary components of fire and maneuver. The computations are described which maintain a sensible sequence of these activities.

The general characteristics of an electronic computer are considered along with the analysis only where necessary. The computer characteristics and capacity for calculation included in this phase of the discussion serve as guide lines for the analysis. Thus the particular way in which the complete combat action is dissected into simpler actions reflects in some measure the types of calculations for which the computer is best suited.

The Military Situation

Col. C. Billingslea* formulated the military situation which might result in a combat action having the characteristics described in the Introduction. It is a hypothetical situation constructed for these special purposes and is not presented as being either a typical situation in some future war nor as representing a typical mission for the troops involved.

A heavily reinforced Blue infantry battalion is given the mission of delaying a Red mechanized Corps, in column, for a period of 12 hours at Münnerstadt, which lies about 30 miles south of the Zonal (East German) boundary at Meiningen on a railroad line to Würzburg (Figure 1). Delay is to be effected by forcing the Red Forces to deploy under heavy fire at the river line which is the northern boundary of Münnerstadt.

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* Military Advisor to ORO during early part of study.

The infantry battalion is supported by a medium tank company of M-48's, a heavy mortar company and a battery of 105mm howitzers. Fig. 2 shows the troop disposition.

Before the battalion had fully occupied its position in and about Münnerstadt, the point of the main Red column approaches and is brought to a halt under fire. The Red point begins to deploy, sending a strong force to cross the river on the right flank of the position. Red combat engineers succeed in quickly erecting a temporary bridge, and a company of 10 T-34's, a company of 5 SP-100's and a company of infantry cross the river and assemble on a hill nearby. They can now bring direct fire on much of the road south out of Münnerstadt along which the Blue forces must soon withdraw. Further, they will quickly attempt to cut that road in an enveloping maneuver.

Fig 1. General Military Situation

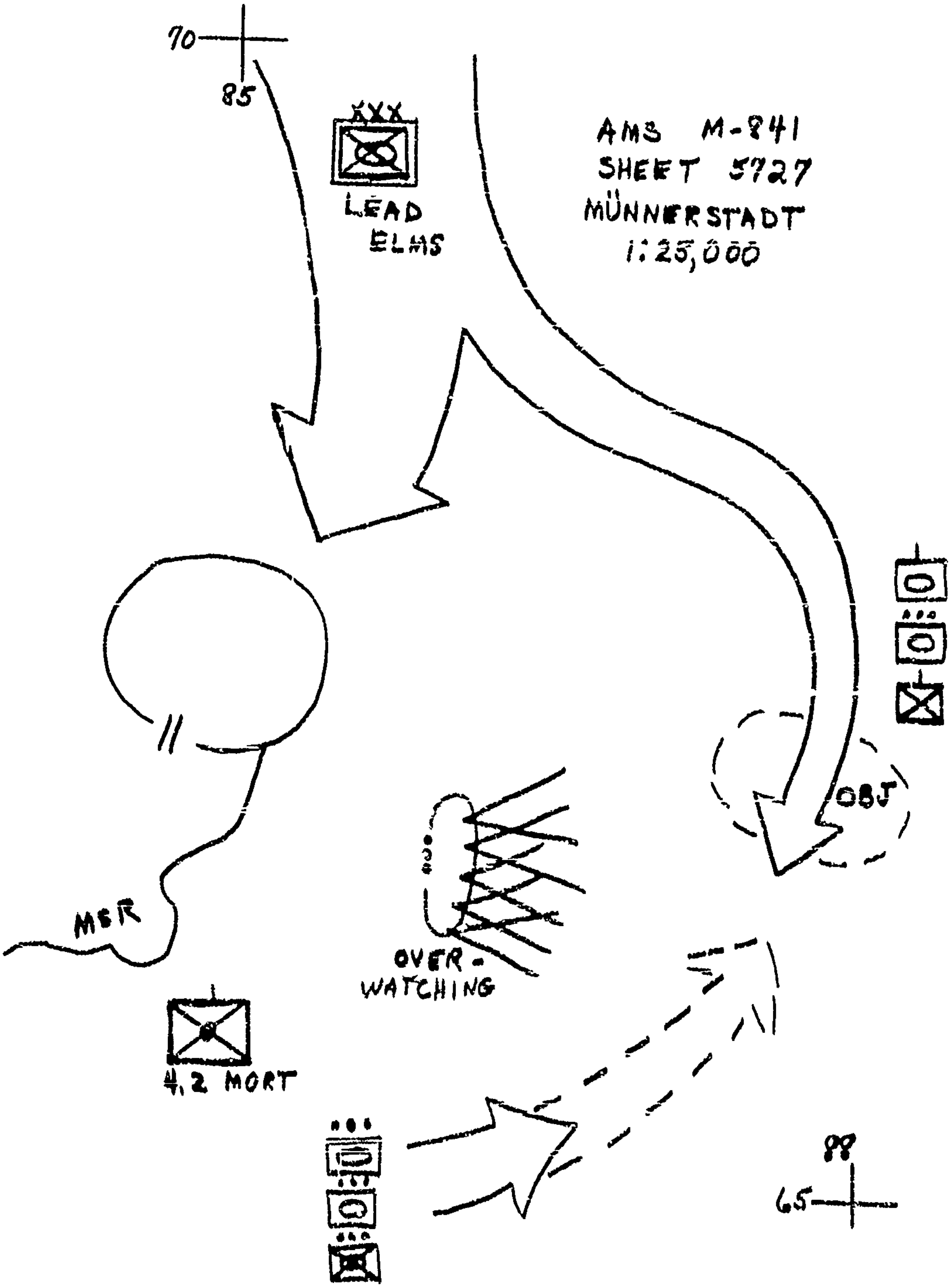

Fig. 2 Disposition of Forces

In the face of this threat, the Blue forces commit their reserve tank company reinforced by 3 squads of infantry mounted in 3 armored personnel carriers (a "scratch" force, since T/O&E does not include carriers). The mission of this force is to push the Red force back across the river in preparation for the withdrawal of the Blue forces in Münnerstadt. The resulting armored assault is the action programmed for the computer.

The Battle

The tactics of the counterattacking Blue force are to provide (1) an assaulting group composed of 2 platoons of M-48's (10 tanks) and the platoon of infantry (3 armored vehicles, one squad each); and (2) a covering force (overwatching) composed of the C.O., F.O., and one tank platoon (total 7 tanks). In addition the company of heavy mortars (4.2 inch) is available as support. The remainder of the Blue force is heavily engaged elsewhere with the Red point.

The assault group moves towards the Red bridgehead, keeping in the draw (Fig. 2) as far as possible, then making a frontal assault. The overwatching tanks provide support fire from cover and concealment at a range of about 1300 yards.

The Blue infantry dismount from their carriers when the Red Position is reached. The Blue Mortar fire is also lifted at this time. The mission of the Blue forces after reaching the Red position is to move on through the Red position, firing as they go. Since the battle must feature intense action lasting a half hour or less, to meet the requirements generated in the Introduction, no further mission for the Blue forces is stated.

Components of the Battle: Fire and Maneuver

The basic actions, which taken together comprise the over-all battle, have long been identified by tacticians with the phrase "Fire and Maneuver." The way in which individual actions of firing or moving are arranged in time and space is the result of the commander's application of doctrine and his own good sense in competition with the doctrine, good sense, and capabilities of the opposing forces.

The necessary sequence of analysis followed by this memorandum is therefore seen to be:

1. Identify the nature of the basic capabilities of the individual combat elements on the battlefield;

2. develop a system for the computer to compute the basic actions on a battlefield from data about the capabilities of the individual combat elements;

3. provide the means for the computer to arrange the possible basic actions of the individual combat elements into a sequence of Fire and Maneuver activities reflecting the sense of any stated tactical doctrine.

This study carries out the above three sotps for the trial battle described only to the extent required for demonstrating the feasibility of the methodology.

Firing the main tank gun appears to be the simpler of the two basic actions when the individual tank is considered as a basic combat element. Given the correct "kill" probability for the circumstances surrounding any particular shot, a Monte Carlo ("coin flipping") decision can easily be made by the computer to determine whether the given round did "kill" its

target. Thus suppose that the correct kill probability for the round is 0.4. Then if the computer chooses a number at random between 0 and 1, there is a 40% chance that the number so chosen will be less than 0.4 and a 60% chance that it will be greater than 0.4. Thus the computer will be using the proper kill probabilities if it makes its decision as to whether the target was killed by the given round, if it "chooses" a number at random between 0 and 1; calling a "kill" if the number is less than 0.4, a miss if greater than 0.4.

There are various ways in which the computer can "choose a number at random."

The above description clearly leaves out most important factors in the "firing" action. In particular, it is also necessary to (1) select a target and (2) decide to fire at the target.

The decision to fire or not to fire at the selected target depends on (1) whether the tank is physically capable of firing — i.e., has a loaded gun which has been laid on the target, and (2) a tactical decision on the desirability of firing at that particular time.

The selection of a target means that the potential targets already picked up by the tank commander are made the subject of a priority system which eliminates all but one of the potential targets.

The above discussion leads to a systematic statement of the time sequence of events in an elemental action of FIRING; at least for a tank

firing its main armament. It is best summarized by the following diagram (called a "flow diagram"):

Step 1		Step 2		Step 3		Step 4		Step 5
Decide whether on firing mission	→	Take note of potential targets	→	Select highest priority target	→	Allow time for loaded gun to be laid on target	→	FIRE by choosing random number and comparing it to kill probability

It will be recognized that this diagram does not allow for all eventualities. For example, it might be argued that something may occur to change the tank commander's mind during Step 4 while the turret is being rotated. Or that Step 1 should follow Step 2, so that the decision to fire depends in part upon what type of targets are available. For the moment, however, the flow diagram just derived will be taken as approximately describing the essential character of the elemental combat action of firing the main gun on a tank. The value of stating this approximate sequence of actions at this time is that certain requirements are generated for the detailed methodology. Thus the computer battle must be capable of providing answers to a series of explicit questions for each tank on the battlefield.

Step 1. Is the tank on a fire mission?

Step 2. What potential targets are known to the tank commander?

Step 3. Which of these targets has the highest priority?

Step 4. How much time before the gun is loaded? Is the gun currently laid on the selected target? If not, when will the gun be on target?

Step 5. What is the correct kill probability for this particular situation?

Knowing that these are (some of) the questions which must be "answered" by the computer provides a key to the types of calculations which will be required of the computer.

The above discussion was specifically pointed at consideration of a tank firing its main armament. However, the computer battle requires that two other types of firing actions also be considered, small arms and artillery fire.

Specifically, small arms fire is delivered by tanks against infantry and by infantry against infantry. Also mortar fire is delivered against infantry.

The mortar fire is treated as a special case in the present battle. Only an "average" treatment is given for the 5 steps outlined for firing the tank gun. The steps are:

1. Point of burst of a salvo of 12 mortar rounds selected at random in the general area occupied by the Red forces.

2. If there are infantry units within 100 meters of this point then a degradation factor is applied to the infantry strength of the unit which is a function of the cover and concealment of this area.

3. The time when the next salvo will be fired is computed.

However, the same systematic treatment of small arms firing is made as is used for the main tank gun.

This is accomplished by considering the small arms fire as being lumped together into discrete units of fire which are delivered at the same rate as the main armament for the tanks and at comparable rates for the infantry units. Since infantry units in the present battle involve more than one discrete fighting unit (more than a single man), on the average, one burst of

small arms fire would not totally destroy the entire combat infantry unit. Instead, it would only reduce the fighting potential of that unit by a fraction. For example, under the proper circumstances, one 30-second burst of MG fire by a tank at an infantry squad might reduce the effectiveness of that squad by 1/4. Determination of the proper fraction involves not only deciding the number of casualties, but also the influence on the effectiveness of the entire squad of such a loss.

With this difference noted the general treatment of firing suggested by the 5 steps will be considered to apply to all combinations of tanks and infantry, with suitable adjustment of the performance characteristics.

Before indicating what system of calculations is used to compute the 5 steps of firing, the list must be extended to include the other basic combat action — MANEUVER, because firing and moving are <u>not</u> independent actions.

It is proposed that the following statement reveals the fundamental character of those separate actions which, when taken together, comprise the whole of MANEUVER. <u>A tank is at the position A on a battlefield. It has the capability of moving to any one of a number of nearby positions, B_1, B_2, ... etc., in some brief interval of time. Formulate the rules which will permit</u> the tank to make a realistic choice among these possible new positions.

A list of some of the factors which must influence such a choice are:

1. Desirability of remaining in present position and firing;
2. direction to terrain objective;
3. whether or not now under enemy fire;
4. character of terrain differences among possible new positions; e.g., swamp, thick concealment, crest of hill, steep slope;
5. presence of enemy fire on neighboring positions.

Thus if these factors are to be taken into account by the computer, a means must be provided for the computer to:

1. Determine character of terrain at the various possible new positions.
2. Have knowledge of terrain objective.
3. Have knowledge of delivery of enemy fire on various positions.
4. Decide whether tank is to move in any event.

Thus the proposed scheme of calculation for computing the outcome of a battle must be capable of answering questions of the type listed under the preceding discussion of the elementary combat actions making up FIRE and MANEUVER.

Again the above proposal can be applied not only to tanks, but also to infantry units. Thus a squad of infantry can be treated on the average as if it too moves from one small area A, to some adjacent area, B_1 or B_2 ... etc. The infantry unit will probably have quite different performance characteristics from a tank. Thus an infantry unit will be able to move over much more difficult terrain than a tank, although with a lower top speed.

Summarizing, it is proposed that the over-all combat action can be considered as comprising the sum total of a large number of elementary combat actions of FIRE or MANEUVER. A systematic statement of the components of these two elemental actions has been proposed. These statements raise specific questions about the locale, participants, and progress of the battle which must be taken into account. Thus specific requirements on the types of Computer calculations which are necessary are generated.

Terrain

Since both FIRE and MANEUVER have been noted as depending strongly upon the terrain factor, neither element of the over-all battle can be further discussed until a means for inserting terrain factors into the machine is selected.

Although there are various alternate means of including factors of terrain we will mention here briefly only the method selected for this feasibility study.

Essentially the choice made is to dissect the battlefield into the largest number of small squares consistent with the capacity of the computer to be used. With the battlefield under consideration this results in each square being 100 meters on a side for a total of 576 squares over the entire battlefield of about 2 square miles.

For each square, the average terrain factors are listed and stored in the memory of the computer. These factors are average elevation, average concealment — in steps of 1/4 from completely open fields to dense forest, plus the presence of selected special characteristics such as swamp, military crest, steep slope, and a road or trail.

The information about the terrain stored in the machine's memory is used by the computer in answering the questions listed in the previous section relating to the calculation of each separate combat action of FIRE or MANEUVER.

For example, Step 2 in the flow diagram for firing (p. 124) requires that the computer "take note of potential targets." One essential factor (but not the only one) is identifying which (enemy) units are in plain view

of the tank attempting to pick up a target. If the elevation of all squares are known, then the computer can determine whether any square between shooter and target is so high as to cut off the view of the shooter. If there is one such square, then that particular enemy unit could not possibly be a potential target. Similarly, if the enemy unit is in the midst of dense forest, then it cannot be seen by the shooter, even if no intervening ground interrupts the "line of sight."

Dissecting the battlefield into squares also serves to make specific the fundamental actions of moving, which, taken together, comprise maneuver. Thus, recalling the statement of the problem of moving proposed in the last section, it may be restated as "A tank is on square A on the battlefield. It has the capability of moving to any one of the eight adjacent squares (p. 136) in some brief interval of time. (It may also remain in its present position, making a total of 9 possible courses of action.) Formulate the rules which will permit the tank to make a realistic choice among these 9 possible courses of action.

Thus it is seen that, if the terrain of the battlefield map is put into the machine's memory in the form of the average terrain features of distinct (small) squares, it is possible to provide approximate specific answers to the type of terrain problems one expects in the course of computing each separate, elementary combat action.

Battlefield Time

Step 4 in the systematic treatment proposed for the elementary combat action of firing, requires that the computer "allow time for a loaded gun to be laid on target." Also computations of the movement of tanks and other combat units require that the proper time be allowed for the combat unit to reach its new position before the computer considers still another change of position. Thus both elementary combat actions require reference to the passage of time in the simulated battle.

An essential difference between the simulation of battle by the method under study here and simulation machinery long used by design engineers is the difference in the treatment of this matter of time. In the more common simulating devices* there is a direct relation between the relative time the computer gives to each section of the calculations and the actual duration of the same processes. Thus a computer designed to simulate the flight of a guided missile would usually compute the curves describing the position of the simulated missile, second by second, just as the missile should actually progress along its trajectory during the same time interval; that is, the "computer time" is the same (within a scale factor) as "real time." This is not the case in the present battle. In the present case, battlefield activity is assumed to be completely stopped while the computer determines what the next situation will be, just as the clock may be stopped during a football game. As soon as the computer has determined the next situation, it immediately skips over all the "real time" actually required for the change to take place and "stops the clock" again while calculating the effects of the most recent change and selecting the next course of action. Thus in this

* Usually ANALOGUE in nature.

simulation of battle, there is no connection whatever between "battlefield time" and "computer time."*

The computer keeps the calculations of the various elements on the battlefield in a proper time sequence by the use of what we will call "alarm clock words" or "clocks" for short. Ignoring for the moment certain complications arising from compromises made in this first coding of the battle, the treatment of time, using the "alarm clocks" is as follows.

Each independent element on the battlefield is assigned a memory location for its personal alarm clock. This clock must have in it the statement of the time in the future at which the associated element expects next to do something; to move, or fire, or look for a target. In order to select the next tank to be processed, the computer looks at all of these "clocks" and finds the one set to the earliest time. It then assumes that time on the battlefield has reached the value of this earliest clock; it examines the situation that the selected combat element finds itself in, makes a decision as to what the element does at this time, how long it will take, if uninterrupted, and finally resets the alarm clock to the time when the element should be considered again by the computer. In the process of consummating the activity of the unit being treated it may have to readjust the clocks of other units on the battlefield. As soon as the computer is through processing one unit, it searches through all the clocks and selects the next unit to be processed. It continues this pattern until the battle is over.

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* This contrast will have to be taken into account if an attempt is made to use this type of battle simulation for a training or research device where the course of the machine calculations is interrupted so that the operators can insert command decisions which have meaning relative to "real time."

Actually in the present battle each unit is provided with two alarm clocks, one governing firing and another movement from square to square. A special set of rules is used to remove, on the average, any ambiguity caused by simultaneous moving and firing. A more reasonable treatment of this factor exceeds the capacity of the 1101 computer.

Analysis of Firing

With the method for calculation of the terrain factor and battlefield time established, it is now possible to complete the analysis of firing begun in a previous section. Recalling that the firing activity by a tank is represented by a systematic, five-step process, the calculation of each step can now be outlined for every tank on the battlefield.

Step 1 involved a (tactical) decision as to whether the particular tank was on a fire mission. This is accomplished in the present study, by a decision made in advance of the actual computations.

All tanks will fire, given a target, as soon as physically possible to do so; except*

1. No firing permitted by assaulting tanks until one of their number reaches edge of Red position;

2. or until the elapse of 15 (battlefield) minutes after start of battle, whichever is earlier.

Step 2 involves those computations which list all potential enemy targets known at the time to the tank commander under consideration. As already indicated, part of this step involves determining which enemy units it is possible for the tank commander to see by reason of cover (elevation) and concealment (foliage). Other factors involved which are treated in varying degrees of completeness are:

1. Which enemy units have disclosed their position by fire or maneuver to any member of the opposing side, together with the chances that all units of either side will share such knowledge through the radio net.

*This limitation was principally a practical one, so as to stay within the time limits on use of the computing machine. When firing was permitted to start with the onset of the assault, the computer calculations consumed an hour per battle, three times too long.

2. Which enemy units have previously been actually noted by the tank commander.

3. Which enemy units are placing fire on the tank in question.

Step 3 involves selecting among the potential targets that one which has the highest priority. The priority system used in the present battle is, from the highest to the lowest:

1. The tank which is firing at the shooter (random choice if more than one).

2. Tank which was last target.

3. Any tank (make random choice)

4. The infantry unit which is firing at the shooter (random choice if more than one).

5. The infantry unit last fired at.

6. Any infantry unit (make random choice).

Step 4 involves establishing that the gun has been reloaded and is laid on the target. Time has already passed sufficient for the gun to have been reloaded and for minor adjustments of the gun's sighting before the tank was selected by the computer for processing. This has been described in the previous section on "Battlefield Time." However, if the target selected in Step 3 is a new target, then an additional time delay is required while the turret is traversed and the gun accurately laid on target. In the present battle, a constant delay of 8 seconds is allowed for this when necessary. In case this delay is required, 8 seconds is added onto the "firing clock" of the shooter and the computer stops computations for the tank. When the tank is selected again for firing, it will then have its

gun laid on target and will be able to fire immediately, unless in the meantime, the target has disappeared from sight, been killed, or if another target of higher priority has become known to the shooter.

Step 5 involves the actual firing. The main problem at this point is to determine the correct kill probability for the particular set of circumstances. The kill probabilities are stored in the computer's memory and depend upon the following 7 factors:

1. Type of shooter (weapon)
2. Shooter moving or not
3. Type of target (armor -- size)
4. Target moving or not
5. Range to target
6. Cover and concealment of target (e.g., hull defilade, in edge of forest).
7. First or subsequent shot by shooter.

The last section of Step 5 carried out 3 calculations:

1. Keeps track of which targets are killed.
2. Readjusts firing clock for shooter's next firing turn
3. Determines whether shooter has disclosed his position to enemy.

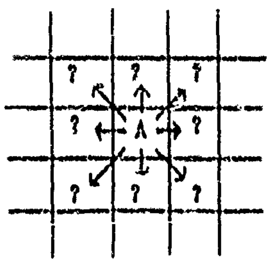
This completes the general description of the basic firing action by tanks. In the case where an infantry unit is doing the firing, the computations are exactly the same, although the interpretation is somewhat altered as has already been discussed. In the event that an infantry unit is the target, Step 5 is altered somewhat. If an infantry unit is within effective small arms range of enemy units (M.G. on tanks or rifles and

automatic weapons of opposing infantry) then the infantry target is taken as almost certainly suffering a few casualties if it is seen. Thus, the "kill probabilities" become for infantry targets, the factor by which the effectiveness of their unit is degraded, rather than the probability of their unit being destroyed. An exception might be for the case when the Blue infantry units are mounted in armored personnel carriers, except that in the present series of battles the infantry dismount before the shooting starts.

Analysis of Maneuver

The general factors which should influence the movement of a tank or infantry squad from square to square on the battlefield have already been discussed. The division of the battlefield into small squares, 100 meters on a side, has been proposed. It remains to describe the specified manner in which the terrain factors, enemy actions, and tactical decisions shall influence the movement of the combat elements.

It will be recalled that the over-all maneuver of the forces is to be considered as resulting from a large number of a simple move decisions made repeatedly throughout the battle for each combat element. Each elementary move decision requires that the computer determine which of the 8 neighboring squares shall be the next position of the combat element in question; also allowing the combat element the option of remaining in its present position.



To do this, each of the neighboring squares is scored separately on its desirable characteristics. For example, one neighboring square might be allowed 25 points if movement to that square is directly towards the terrain objective. Another square might be given a score of only 5 points if movement to that square is off to one side of the terrain objective, and a square which is on the opposite side of the present position away from the terrain objective would be scored as zero, or even negatively, so far as contributed toward reaching the terrain objective is concerned.

Thus a series of scores -- or ratings -- is adopted which is to be associated with each square on various accounts of its possessing desirable or undesirable terrain features, or exposure to enemy fire, or any other factor which is thought to contribute to the desirability of movement to that square. By totaling up all the individual scores on different accounts for each square, a number is associated with each square which is the higher, the more desirable movement to that square at that particular time appears to be, at least insofar as the tank commander can determine. At this point it would be possible to have the computer select as the next position for the combat unit, that square which has the highest rating. <u>It is of the most basic significance that this has not been done in this feasibility study</u>. It is essential that the reasoning behind rejecting this possibility receives the most careful attention.

There are several different reasons for rejecting the above proposal, -- and several different ways of looking at those reasons. One way of putting it is to assert that <u>all performance data to be inserted into the battle must be capable of being determined by field experiments or by a study of history</u>.

But it should be clear that, were a number of different tank commanders put in the same position on the battlefield, under identical circumstances as far as they could be determined, all the men would not choose the same square (or even the same general direction) as their next position. Yet if the computer always chooses that square which acquires the highest rating, throughout many different battles, it would be asserting that all men would do the same. Thus, the ratings could not be completely determined by experiment, not even in principle, since in the experiments there would surely be some variation in choice among different men.

Another way of looking at the same problem is to consider what would happen if there were 2 squares in quite different directions which had nearly the same total rating, e.g., differed by only 1%. If the computer always chose the square with the highest rating, then this is tantamount to asserting that the rating numbers are so accurately known that it is reasonably certain which is the more desirable. It would seem to be overly optimistic to assert that experiment in (or history of) such a complex matter could ever produce answers with such certainty.

A third way of looking at this matter is to consider whether it may be important to determine the influence on the outcome of a battle of various assumed degrees of variability in the responses of men to the same situation. Thus it might be argued that weapon system A is better than weapon system B because A functions better with men who have received only 6 months of training than does B, although if all men could receive 6 years of training B would be the better choice. In other words, the extent of the variation in the response of different vehicles to the same situation might be considered as related in part to the thoroughness of training.

Each of the three points of view presented above points towards the inadequacy of the system where the computer always chooses that square which acquires the highest rating. The simplest alternative to such a rule is to cause the computer to interpret the rating numbers as the relative probabilities with which the combat element will choose its next position six of the adjacent squares. This is what is done in the present study.

On the other hand, there will undoubtedly be some situations where it is desirable to remove even a slight chance of moving into some particular square. This is accomplished in the present battle by allowing negative ratings to be assigned for certain special situations. If these negative values are made large enough they can certainly cancel out any possible positive score the square might acquire from other considerations. The computer then is instructed to consider only positive ratings as a valid relative probability, hence there is no chance of selecting that (negative value) square.

There is also the possibility of suspending the entire rating process in emergency cases and making selection of a particular square a certainty. This has been done in the present battle for the special case where a tank has just moved from a covered (or concealed) position and has been fired on. In this case the tank always returns to the covered position.

Thus the methodology is flexible enough to permit considerable modification of the maneuver calculations should that prove desirable for special cases.

Summarizing, a military situation has been described which results in a small combat action meeting the requirements developed in the introduction. The combat action itself has been dissected into small pieces of terrain and combat actions. A series of precise calculations and decisions have been proposed which, taken together, are a systematic means for calculating the outcome of each separate elementary combat action of FIRE and MANEUVER. Finally, a system for keeping track of the passage of battlefield time has been described which will permit the computer to keep a sensible sequence in the order in which the separate elementary combat actions are computed.

RESULTS OF TEST BATTLES

Results of the trial calculations are required for two purposes:

(a) To establish the spread* of battle outcomes deriving from the nature of the model of battle, and from the spread of results to assess the statistical reliability of average battle results.

(b) To establish the sensitivity of the average battle outcome to a significant alteration in the performance characteristics of the Blue tanks only.

Once these two parameters are determined it is possible to specify the number of repetitions of the battle that are required to indicate, for instance, the better of the two tank designs.

The principal results of the trial calculations are applied in this section to this determination.

Spread of Battle Results

The most basic characteristic of the model of battle described in this memorandum is the influence of the play of chance that is included. Figure 3 shows the variation in the number of tank casualties suffered by the Blue side, equipped with medium tanks, in 50 battle calculations that differed only by virtue of the play of chance. This figure also shows the variation in Red Tank losses (T-34's and SU-100's) during the same 50 battles. Although on the average Red suffered 7.1 tank casualties per battle compared to Blue's average losses of 10.4, it is evident there were many departures

* "Spread" as used here is equivalent to the standard deviation of the distribution. For normal distributions this is the interval about the mean which includes 63 percent of the cases. Table 1 gives the spread of all the casualty distributions presented.

from this average. Figure 5 shows that in 6 of the 50 battles the Red losses were actually larger than the Blue losses. This fact is indicated in Figure 5 by the 6 points above the dashed line, along which the losses on both sides are identical.

If the number of battles were increased beyond 50, the spread in tank losses indicated by Figure 3 would in all likelihood not be changed significantly. There is only 1 chance in 1000 that it should vary by more than plus or minus 30 percent. Hence the degree of spread in the results is mainly characteristic of the battle model and the performance characteristics of the man-weapon teams alone.

Testing Competing Tank Designs

The important corollary to the spread in results effected by any given weapon design is the concomitant number of times the battle computations must be repeated to reveal differences among competing tank designs.

To investigate this feature of the methodology, 50 additional battles were computed for the case where the Blue medium tanks were replaced by the same number of hypothetical light tanks. All other features of the battle situation remained as before. Figure 6 shows the distribution of the number of tank casualties experienced by both sides in this second series of battles. On the average, Red lost 8.4 tanks in each battle, whereas Blue lost an average of 6.5 light tanks per battle. Thus, based on the average number of tank casualties alone, the Blue hypothetical light tank was more effective than the Blue medium tank. In particular the average effectiveness ratio* for the

* A simple definition of tank effectiveness has been used by V. McRae and A. Coox in ORO-T-278, "Tank-vs-Tank Combat in Korea." There, tank effectiveness was defined as the ratio of the average number of enemy tanks killed by each friendly tank to the average number of friendly tanks killed by each enemy tank. Other definitions of effectiveness have been proposed, including cost effectiveness, which includes the elements of production and logistical costs.

Blue medium tank battles was 0.6 (to the disadvantage of Blue) whereas for the hypothetical Blue light tank the effectiveness ratio was 1.14 (to the advantage of Blue).

It is at this point that the degree of spread in the number of tank casualties in the various battles must be considered. The two effectiveness ratios 0.61 and 1.14 calculated above are statistical approximations to the "correct" values that would have been produced had the battle computations been repeated an "infinite"* number of times. Thus there is always the chance, however remote, that both these numbers are so much in error that, in fact, the Blue light tank is actually less effective than the Blue medium tank. It is possible to reduce the risk that such an erroneous conclusion would be drawn to any size however small, at the expense of increasing the number of test battles.

Application of standard statistical tests on the reliability of these test results shows that the odds are overwhelming against (better than 360,000:1) the possibility that either one of the two series of 50 battles incorrectly identified the winning side.

The conclusion is that a sample size of 50 battles was sufficient to demonstrate the superior killing powers of the Red tanks in this series of battles. Indeed, a substantially reduced number of repetitions would probably have been acceptable. Figure 4 shows what the average losses for the Blue medium tanks would have been had the battle calculations been stopped after each of the 50 battles in turn. From this figure it is found that the

* For practical purposes, "infinite" can be taken to mean a very large number, e.g., 1,000,000.

computed effectiveness ratio varies by only about ±3 percent as the number of battle computations is increased beyond 30. It is evident that sequential sampling techniques may be applied to minimize the quantity of calculations.

The previous discussion does not require that the distributions of tank losses shown in Figures 5 and 6 be normal. However, in view of the unusual character of the distribution of Red casualties shown in Figure 3, a test on the statistical hypothesis that each of the four distributions was normal gives the results shown in Table 1. The results show that all four distributions are well within the 0.05 level of significance.* If there were serious concern regarding whether these distributions may be approximated by normal error curves, then an appeal to statistical rigor could only be supported by the results of additional computer calculations.

TABLE 1

STATISTICAL TEST ON THE SIGNIFICANCE OF OBSERVED DEVIATIONS FROM NORMAL ERROR CURVE FOR FOUR DISTRIBUTIONS OF TANK CASUALTIES

Category	Mean losses	Standard deviation	Probability of observed departure from normal curve by chance alone
Blue medium	10.42[a]	2.33	0.95
tank battles	7.08[b]	2.74	0.35
Blue light	6.46[c]	2.38	0.21
tank battles	8.36[d]	1.66	0.29

[a] Blue (Fig. 3) [c] Blue (Fig. 6)
[b] Red (Fig. 3) [d] Red (Fig. 6)

Fourteen additional battles were computed for the case where the Blue forces were equipped with a hypothetical heavy tank. The Blue forces were

* So long as the probabilities are greater than 0.05 that the observed deviation from a normal curve could be due to chance alone, the assumption that the distributions are normal is tenable.

the winners in terms of casualties in this series of battles, losing an average of 5.4 tanks per battle compared to the average Red losses of 8.8 tanks per battle. The sample size of 14 is so small as to cast doubt on the reliability of the results however.

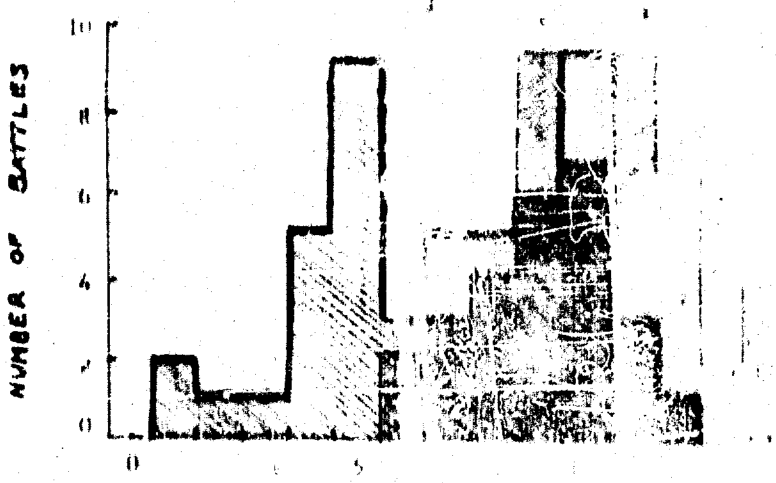
The conclusion is that a series of 50 battle calculations for each tank design may be expected to be sufficient to identify the superior tank design features in the present instance when significant variations in major tank design features are assumed.

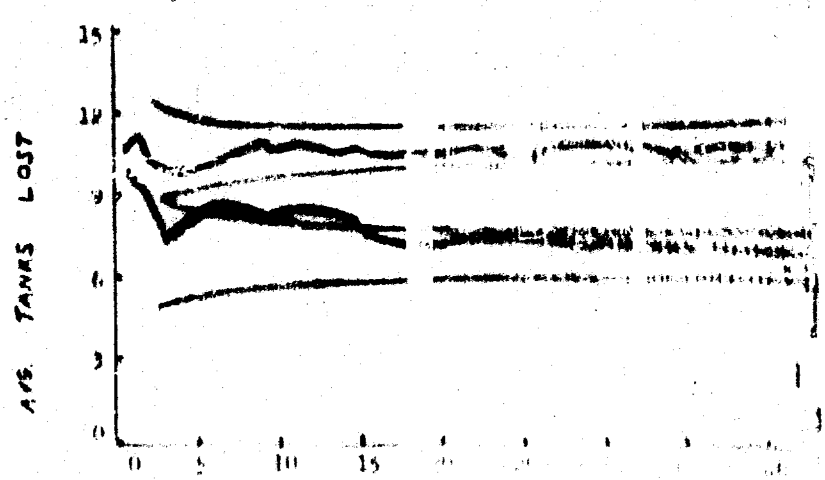
Discussion of Results

It must be emphasized that the superiority is stated only in terms of some battle result that it has been agreed will indicate superiority. Clearly there are different aspects of superior performance. For example, in the preceding calculations, relative tank killing power has been used as indicating superiority. Other factors could have been used in its place. Thus, superior Blue performance could have been measured solely in terms of the destruction of the Red forces regardless of the Blue losses sustained in the attack. Or superior Blue tank performance could have been taken as being indicated solely in terms of the number of Blue tanks that were able to reach the terrain objective. Or any combination of these features could have been used to measure superior performance. The purpose of this feasibility study is not to fo[illegible] the criteria of superior performance but to provide the means for simulating battle so as to permit identification of superior performance once it has been defined.

Conclusion

The Monte Carlo technique enables a very large number of battle factors to be introduced into a feasible analysis of the performance of alternative weapons and weapons systems. The number of battle factors warrants designation of the computing system as a battle simulator.


NUMBER OF BATTLES
0
5


AVG. TANKS LOST
15
0
0
5
10
15